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Identification of the selected parameters of the model in the process of ballistic impact

K. Jamroziak*
Faculty of Security Sciences, Military Academy of Land Forces,
ul. Czajkowskiego 109, 51-150 Wroclaw, Poland
* Corresponding author: E-mail address: krzysztof.jamroziak@wso.wroc.pl

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Analysis and modelling

ABSTRACT

Purpose: Analysis of the process of overshooting the material with high speed refers to the identification of certain properties of elasto-dissipative materials. The result of this identification is to determine the value of deformation on the basis of changes in the speed of the projectile inside the material until it stops or overshoot this material.

Design/methodology/approach: On the basis of the proposed dynamic models of piercing the material using energy balance equations, dissipation of the energy of mass which strikes the shield has been described.

Findings: Dependence of the values of elastic energy and dissipative energy has been derived based on the energy balance equations whose values determine the sensitivity of the analyzed parameters of the dynamic models of the overshooting process.

Research limitations/implications: Dynamic models have been analyzed and the impact energy balance equations have been derived. Those equations were the basis to determine the constants and to show their mathematical and graphical interpretation.

Practical implications: Derivation of the dependence for the constants, which are characteristic for the energy balance equations, allowed to describe by dependencies the selected parameters of the model, whose identification may be performed using a special quasi-statistical tests on special stand or in the manner as described.

Originality/value: Presented work including the identification of piercing the ballistic shield is a part of work on the implementation of the degenerated models to describe these phenomena.

Keywords: Computational mechanics; Impact load; Degenerated model; Composites

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1. Introduction

Modeling of dynamic processes is largely an art that requires very good knowledge of the researcher. Modeling of mechanical systems/mechatronics, which is used to simplify too much leads to undesirable results, resulting in such a model at the outset bear is a big mistake. By joining the work associated with adaptation to specific models described in the work [1,2,3,4,5,6,7,8] need to be analyzed in terms of their structure and adopt a capabilities such as the impact energy dissipation. Considerable attention and presents the possibility of degenerate models described in the work [9].

Practical application of degenerated models in the process of piercing materials used to build ballistic shields encounters a number of theoretical difficulties [10,11,12]. This problem is related to insufficient knowledge of the dynamic behavior of these models, which are generally derived from the adoption of complex, nonlinear, constitutive dependencies of materials or from combining rheological models of Maxwell type with linear elastic or dissipative elements. In engineering problems, besides effects of rheological forces, there is always element of mass, which causes forces of inertia in conditions of dynamic loading. Including these forces in the equations of dynamic leads to the necessity of the analysis of degenerated system, which is the subject of many studies [13,14,15,16,17].

Works in this area tend to seek reliable dynamic models in the field of degenerated systems. This approach to searching for description of material properties seems to be appropriate.

The dynamic development of materials science [18] in recent years has generated a lot of modern composite materials. To describe the properties of materials, especially those intended for ballistic protection, non-classical models should be seek. With the use of these models it will be possible to describe, by differential equations, the reaction of the material on piercing projectile.

However, due to the complex structure of the system, assumptions which describe the resisting forces of the pierced material on the projectile, will take an extensive form. We should strive to simplify the description of complex structures of overshooting process, because this phenomenon can be explained by using degenerate models by simple non-classical dynamic models.

2. Dynamic models and their analysis in terms of energy balance equations

Analysis of the phenomena of absorbing the impact energy of small arms projectile is a complex issue of impact load of light ballistic shields. Literature includes a variety of possible solutions to the problem expressed by energy dissipation model [19,20,21,22,23]. Previous analysis contained in the papers [24,25,26,27,28,29] come down to the local issues of the direct impact, and the waves and vibrations that appear in the distance from the place of impact on the shield, do not affect the motion of a projectile in the shield. They appear only after projectile stops or leave the shield. The subject of this analysis is the process of energy absorption on the example of aramid ballistic shields, shoot by 9mm Parabellum bullet type. This analysis has been performed by two independent methods. Each of them uses a different dynamic model.

The first case assumes that the impact of the shield's material on the projectile can be described by internal force in the form of the Kelvin's model with a dry friction (Fig. 1). In the second case the Zener's model with a dry friction has been adopted (Fig. 2).

In both cases the models of reaction between forces of the shield's material and the projectile are determined by the mechanical energy losses and two parameters: *k, h*. However, differences in the structure of these models substantially affect the results of the analysis of energy dissipation.

In both cases, the following assumptions have been made:

a) tracking the movement of the projectile in the shield will allow to determine the material constants of the shield,
b) during movement of the projectile in the shield, the material constants:

- c, k, h (model - 1);
- c, c_0, k, h (model - 2);

do not depend on the projectile's speed and its location.

Study on the mechanical properties of materials show that the second assumption seems to be debatable, since the tests of stretching materials at different speeds shown the dependence of obtained curves of *load-deformation* from the selected speeds [30,31]. Material constants appearing in the models (1) and (2) should be considered as an average values in the range of speed from v_0 (impact speed of the projectile) to the small or even zero speed (stoppage of the projectile in the shield).

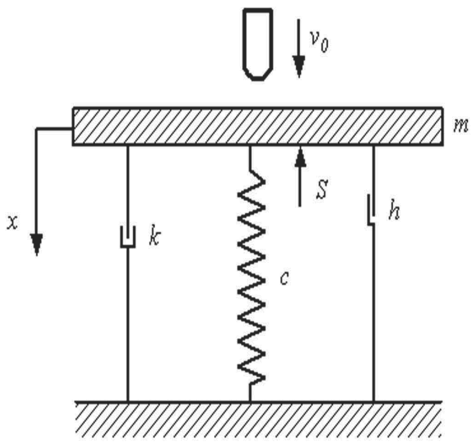

Fig. 1. Dynamic Kelvin's model of strength of the material's shield reaction on projectile

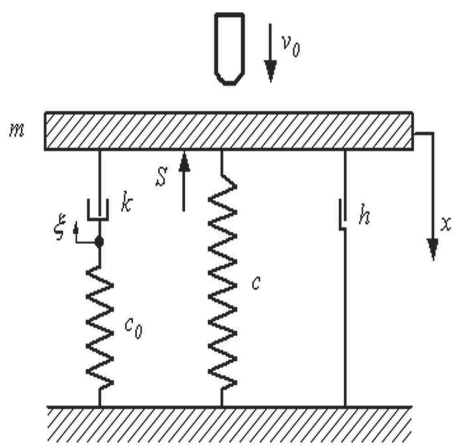

Fig. 2. Dynamic Zener's model of strength of the material's shield reaction on projectile

2.1. Analysis of the absorption of impact energy in the Kelvin's model

According to Kelvin's model (Fig. 1) the strength S of the resistance to motion of the material which depends on projectile's position x in the material and its velocity $v=\dot{x}$ would be describe in the following way:

$$S(x,v)=cx+k\dot{x}+hSgn(v) \tag{1}$$

Differential equation of the projectile's movement would take the form:

$$m\ddot{x}+k\dot{x}+hSgn(\dot{x})+cx=0 \tag{2}$$

Assuming the time interval for $t=0$ (the moment of impact) until the time t_k (moment of the stoppge of the projectile), it is noted that for:

$$t\in(0,t_k) \rightarrow \dot{x}>0 \text{, therefore } Sgn\dot{x}=1 \tag{3}$$

Equation (2) for the time interval (3) would take the form:

$$m\ddot{x}+k\dot{x}+h+cx=0 \tag{4}$$

with initial conditions in the form:

$$x(0)=0, \qquad \dot{x}(0)=v_0 \tag{5}$$

where: moment t_k satisfies the equation:

$$\dot{x}(t_k)=0 \tag{6}$$

The solution of equation (4) is function that shows a damped vibrations with the average value $-\frac{h}{c}$ presented in graphical form in Fig. 3.

Equation (4) has been multiplied by the elementary displacement $dx=\dot{x}dt$ and integrated in the range $t\in(0,t_k)$ to receive:

$$\int_0^{t_k}\ddot{x}\dot{x}dt=\int_{v_0}^{v(t_k)}vdv=\frac{v^2}{2}\Big|_{v_0}^{0}=-\frac{v_0^2}{2} \tag{7}$$

$$\int_0^{t_k}\dot{x}\dot{x}dt=\int_{x_0}^{x_M}vdx=\alpha \tag{8}$$

$$\int_0^{t_k}\dot{x}dt=\int_{x(0)}^{x(t_k)}dx=x\Big|_0^{x_M}=x_M \tag{9}$$

$$\int_0^{t_k}x\dot{x}dt=\int_{x_0}^{x(t_k)}xdx=\frac{x^2}{2}\Big|_0^{x_M}=\frac{x_M^2}{2} \tag{10}$$

After summing up all the terms according to equation (4) we obtain the dependence of the form:

$$-\frac{mv_0^2}{2}+k\alpha+hx_M+c\frac{x_M^2}{2}=0 \tag{11}$$

Dependence (11) shows the energy balance in the interval of time from an impact of a projectile to its stoppage in the material. Impact energy $\frac{mv_0^2}{2}$ (kinetic energy of the projectile) is transformed into potential energy (purely elastic deformation energy) $\frac{cx_M^2}{2}$ and dissipative energy E_d which is described by the dependence:

$$E_d=k\alpha+hx_M \tag{12}$$

The value of dissipation energy depends on the constant k of viscous damping and constant h that describes dry friction. Constant α represents a field limited by a fragment of the phase trajectory $v(x)$ of damped vibrations (Fig. 4).

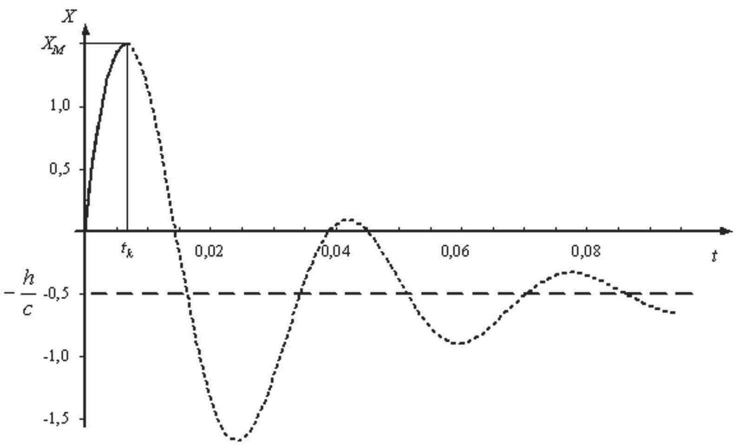

Fig. 3. Graph of the function x(t) - solid line Marks the interval $t\in(0,t_k)$

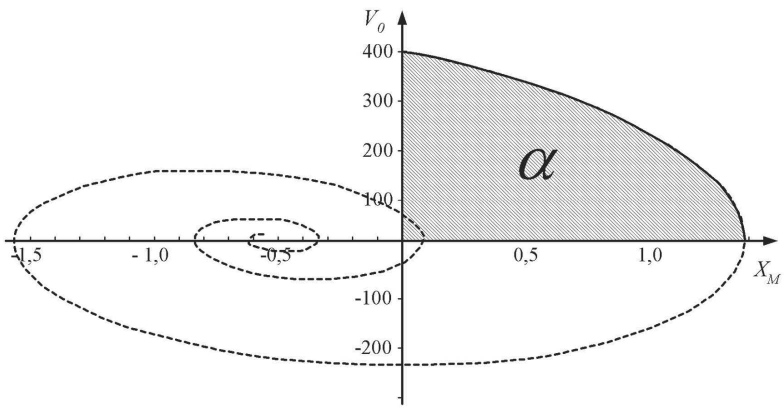

Fig. 4. The graphical form of the constant α of energy balance equation

2.2. Analysis of the absorption of impact energy in the Zener's model

In the Zener's model (Fig. 2) the strength S of the resistance to motion of the material which depends on projectile's position x in the material and its velocity $v = \dot{x}$ would be describe in the following way:

$$S(x,v) = cx + c_0(x-\xi) + hSgn(v) \tag{13}$$

Differential equation of movement of mass m is described by two equations:

$$m\ddot{x} + c_0(x-\xi) + cx + hSgn(\dot{x}) = 0 \tag{14}$$

$$c_0(x-\xi) = k\dot{\xi} \tag{15}$$

Energy losses of impacting projectile should be analyzed in the time interval $t \in (0, t_k)$, so up to the total deceleration of the projectile where the following initial conditions are assumed:

$$t \in (0, t_k) \rightarrow \dot{x} > 0 \text{ to } Sgn\dot{x} = 1 \tag{16}$$

Eliminating the constant ξ from equations (14) and (15) results in reducing dependence to just one cubic equation in the form:

$$\frac{km}{c_0}\dddot{x} + m\ddot{x} + k\frac{(c_0+c)}{c_0}\dot{x} + cx + h = 0 \tag{17}$$

Equation (17) has been multiplied by the elementary displacement $dx = \dot{x}dt$ and integrated in the range $t \in (0, t_k)$ under which we received:

$$\int_0^{t_k} \dddot{x}\dot{x}dt = \ddot{x}\dot{x}\Big|_0^{t_k} - \int_0^{t_k} \ddot{x}\ddot{x}dt \tag{18}$$

Assuming the following initial conditions for $t = 0_+$ and $t = t_k$, so $\dot{x}(t_k) = v(t_k) = 0$ and $\dot{x}(0) = \alpha(0) = 0$ equation (18) takes the form:

$$\int_0^{t_k} \dddot{x}\dot{x}dt = -\int_{v_0}^{0} adv = \int_0^{v_0} adv = \beta \tag{19}$$

The remaining integrals would take the form:

$$\int_0^{t_k} \ddot{x}\dot{x}dt = \int_{v_0}^{v(t_k)} vdv = \frac{v^2}{2}\Bigg|_{v_0}^{0} = -\frac{v_0^2}{2} \tag{20}$$

$$\int_0^{t_k} \ddot{x}\dot{x}dt = \int_{x_0}^{x_M} vdx = \alpha \tag{21}$$

$$\int_0^{t_k} \dot{x}dt = \int_{x(0)}^{x(t_k)} dx = x\Big|_0^{x_M} = x_M \tag{22}$$

$$\int_0^{t_k} x\dot{x}dt = \int_{x_0}^{x(t_k)} xdx = \frac{x^2}{2}\Bigg|_0^{x_M} = \frac{x_M^2}{2} \tag{23}$$

Summing up the dependences (20-23) by equation (17) we obtain the following dependence:

$$\frac{km}{c_0}\beta - \frac{mv_0^2}{2} + k\frac{(c_0+c)}{c_0}\alpha + \frac{cx_M^2}{2} + hx_M = 0 \tag{24}$$

Dissipative energy is so in this case described by the function of a form:

$$E_d = \frac{km}{c_0}\beta + k\frac{(c_0+c)}{c_0}\alpha + hx_M + \frac{cx_M^2}{2} \tag{25}$$

The parameter β shows the field limited by a trajectory $a(v)$ of the vibrations damped in the scope $v \in (v_0, 0)$ shown in Fig. 5.

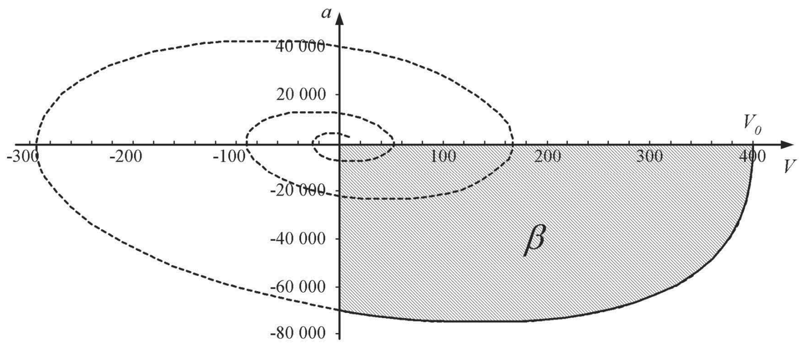

Fig. 5. The graphical form of the constant β of energy balance equation

3. The identification of selected parameters of the model

The energy taken by the shield in the moment of impact, can be represented (in a large simplification) as the sum of elastic energy and energy of dissipation.

$$E = E_s - E_d \tag{26}$$

In order to determine the energy of dissipation for the analyzed system, it is necessary to determine x_M, which value depends on the speed of the projectile in the material. This value, which is defined as α, is expressed by relation:

$$\alpha = \int_{x_0}^{x_M} v dx \tag{27}$$

A linear increase of deformation of the shield has been assumed (which is a big simplification) and for this condition we obtain the relation:

$$v(x) = v_0 - \gamma x \tag{28}$$

where: $x = x_M \rightarrow v_0 - \gamma x_M = 0$, so the constant γ is expressed by relation:

$$\gamma = \frac{v_0}{x_M} \tag{29}$$

Based on the relation (27) constant α can be written in the form:

$$\alpha = \int_0^{x_M} (v_0 - \gamma x)dx = v_0 x - \gamma \frac{x^2}{2}\Bigg|_0^{x_M} = v_0 x_M - \gamma \frac{x_M^2}{2} = \frac{v_0 x_M}{2} \tag{30}$$

identifying constant β which form can be described by the relation:

$$\beta = \int_0^{v_0} a dv \tag{31}$$

where:

$$v(x) = v_0 - \gamma x = a$$
$$a = \dot{v} = \frac{d}{dt}(v_0 - \gamma x) = -\gamma \frac{dx}{dt} = -\gamma v \tag{32}$$

Based on the relation (31) constant β can be written in the form:

$$\beta = \int_0^{v_0} -\gamma v dv = -\gamma \frac{v^2}{2}\Bigg|_0^{v_0} = -\gamma \frac{v_0^2}{2} \tag{33}$$

After substituting equation (29) into equation (33) constant β would ultimately take the form:

$$\beta = -\frac{v_0^3}{2x_M} \tag{34}$$

If we know the mathematical description of the parameters α and β which are characteristic for the energy balance equations, to estimate them we need only to identify the changes of projectile speed in the material x_M.

Estimation of speed changes in the material is possible by measuring the decrease of the speed on the test stand. Identification of x_M can be performed in several ways using statical or quasi-statical tests. However, verification with the use of ballistic test gives reliable results, and involves measuring the decrease of speed in the material by measuring the initial speed of v_0 and missile velocity v_k until the stoppage of the projectile in the material. In this way the form of $v_o(x)$ is determined as it is illustrated in Fig. 6.

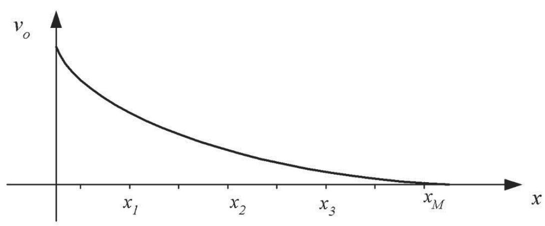

Fig. 6. Method of determining the parameter x_M from relation $v_o(x)$

Definition of constants α and β, which relation has been shown in the form:

$$\alpha = \frac{v_0 x_M}{2}; \quad \beta = -\frac{v_0^3}{2x_M}$$

and determination of parameter x_M comes down to determining the elastic strain energy and energy of dissipation by using equations (12) and (25). Elastic strain energy (see 2.1) depends on the constant c. Identification of the constant c can be easily performed in the tests of quasi-statical piercing in the scope of the unstable deformation. This relation can be obtained on the basis of experimentally obtained relationship diagrams $\hat{S}'(x)$ and $\hat{S}''(x)$ (Fig. 7) in the scope of the unstable deformation.

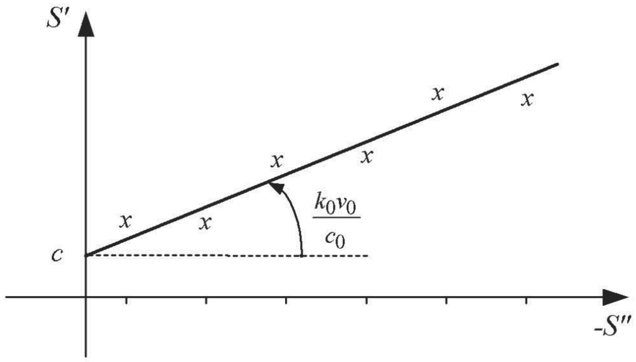

Fig. 7. Method of estimating the parameters c, k_0/c_0 based on the determination of $S'(S'')$ in quasi-statical test

The value of energy of dissipation depends on the constant k of viscous damping and constant h that describes the dry friction. These parameters are responsible for the behavior of the material under conditions of permanent deformation. They can be determined on the basis of static tests involving loading the material of the shield by constant forces S_0 larger than the constant h, while measuring the speed and acceleration of deformations. In earlier works [15,32] permanent deformation has been defined in a form:

$$x(t) = u(t) + \xi(t) = \frac{S_0}{c}\left(1 - e^{-\frac{c}{k_z}t}\right) + \frac{S_0 - h}{k}\cdot t \tag{35}$$

After differentiating the above function we will receive the speed of the form:

$$v = \dot{x} = \frac{S_0}{k_z} e^{-\frac{c}{k_z}t} + \frac{S_0 - h}{k} \tag{36}$$

and the acceleration will take the form:

$$a = \dot{v} = \frac{-S_0 c}{k_z} \cdot e^{-\frac{c}{k_z}t} \tag{37}$$

After eliminating the parameter t from both equations, we obtain a relation:

$$S_0 = h + k\left(v + \frac{k_z}{c}a\right) \tag{38}$$

By using the method of regression analysis, determination of values of constants h and k leads to the identification of relation of variable $Z = v + \frac{k_z}{c}a$ $Z = v + \frac{k_z}{c}a$ in a function of different values S_{0i} (i = 1, 2…n) (Fig. 8). The value of the substitute damping should be previously determined based on the relation:

$$k_z = k_0 \cdot \frac{c + c_0}{c_0} \tag{39}$$

where replacement value of the damping k_z.

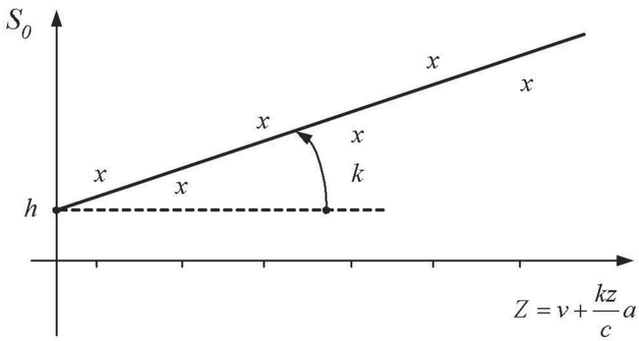

Fig. 8. Identification of constants h, k based on the relation $S_0(Z)$

4. Conclusions

The paper presents the identification of the dissipation energy of the projectile impacting the shield made of composite materials. The analysis of selected dynamic models has been performed by using the equations of energy balance. A common part for both models adopted for this analysis, was a description of the energy losses by parameters of dry friction and viscous damping. The second model (Fig. 2) belongs to a group of degenerated models. The analysis derived depending on the energy dissipation of energy balance equations are significantly different from each other. In the model (Fig. 1)) this energy depends only on the parameters k and h due to changes in the speed of the projectile in the material through parameter $x_{M.}$ The second model (Fig. 2) takes also into account in its construction the parameters of static and dynamic stiffness of the shield, which are characteristic for non-destructive deformation of the shield. Description of the dissipative energy in the second model, seems to be more accurate. The main reason for this is that it takes into account the parameters of the material in the scope of unstable deformation, especially hysteresis of the material. It is important because it moves the limit of unstable deformation of the shield so, on the basis of work [16] the boundary energy which determines the movement into the destructive phase, reaches its maximum value. The second part of the paper focuses on the identification of selected parameters of the model and provides derived relations for their determination. It can be noticed that the determination of the parameter x_M requires the special testing, which are the goal of the author of this paper. The values of other parameters can be determined through statical and quasi-statical tests that use the method of regression analysis. Because of that, the problem reduces itself to determine the value of deformation of overshoot shield.

The further work should focus on determining the parameters of the model through determining the fields of loops in simulations and then determining the values of parameters of the analyzed models by use of linear regression. In the case of the second model these method of identification (energy balance method) causes many difficulties due to a complex equations, in which the coefficients appearing in the equation of identification are the complex expressions of stiffness, damping and masses occurring in the system. Further work continues towards the adaptation of procedures based on energy balance equations and the balance of power to identify the parameters of adopted degenerated models.

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