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Basic features of the laser acceleration of charged particles

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ABSTRACT

Purpose: of this paper is to study the acceleration of the charged particles by the laser beam in the range outside the resonance conditions. The studies have been limited in the subresonance region since in order to achieve the resonance acceleration a very high constant magnetic field is needed.

Design/methodology/approach: The studies are carried out using the analytical derivations of the particles dynamics and its kinetic energy. The evolution of the acceleration process in time has been studied. The presented illustrations enabled interpretation of the obtained equations.

Findings: The kinetic energy of the particle periodically achieves the maximal energy. Its value and the distance between the subsequent maxima rise with the increasing magnetic field or the laser beam intensity. However, these parameters of oscillating energy decrease with the decreasing wavelength.

Research limitations/implications: Limits in the energy of accelerated particles are caused by the limits of the available at present the laser beam energy and the static magnetic field intensity.

Practical implications: The authors of this paper believe that the presented results of the studies will help the designing of the experimental studies. It has been shown the way of achieving the high energy particles without the application of a very high magnetic field.

Originality/value: The value of the paper is the analytical derivation of the parameters describing the oscillatory shape of the particles energy and numerical analysis its course. According to the authors best knowledge there are no performed such analysis of the acceleration process.

Keywords: Acceleration of charged particles; Laser; Relativistic dynamics; Kinetic energy of a particle

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1. Introduction

The interest in practical development of accelerating devices for charged particles recently is rapidly growing. The elaboration of new theoretical models and many successful experiments resulting in gaining by the charged particles the relativistic energies gave an additional impulse for these studies. This direction of investigation is so interesting because of considerable progress in high power laser construction which creates possibility of the charged particles acceleration to high relativistic energies. No doubt these accelerating devices begin to be alternative accelerators to date used.

There is some evidence of existing the laser based accelerating devices capable of generating narrowly shaped beams of charged particles moving with relativistic velocity of electrons, positrons, and also particles of higher masses as protons and ions. Further progress in this subject can make an essential influence in many areas of technology and science [1-12].

During the last few years, the possibility has been shown of particles acceleration as a result of their interaction with the laser or maser beams of different polarization [13-24] and additionally applied static or pulsating magnetic or electric fields having different intensity and direction [25-34]. Now there are accessible lasers showing radiation power density of the level 10^{22} W/cm^2, which corresponds to the electric field intensity amplitude 10^{14} V/m. This has triggered conduction of many new researches [35-37]. It was found that the frequency chirping [38-43] plays an important role to enhance the electron energy [44-45]. The main purpose of this paper is to show the electron trajectories under various parameters of electromagnetic and magnetic fields of laser beam and to find the conditions at which the particles can be accelerated to large energies under interaction with the laser radiation and the static magnetic field. Another purpose is to indicate the significance of the initial velocity of the charged particle in the acceleration process. The main purpose of this paper is to find the conditions at which the particles can be accelerated to large energies under interaction with the laser radiation and the static magnetic field with the intensity achievable today. The main study is carried out within the range outside the resonance conditions. The studies have been limited in the subresonance region since in order to achieve the resonance acceleration a very high not achievable today constant magnetic field is needed. The studies are carried out using the analytical equations derived by the authors. The presented illustrations enabled interpretation of the obtained equations. The results of the studies will help the designing of the experimental studies. We are sure that subresonance acceleration may be interesting region from the academic point of view and application possibilities as well. We made an attempt to show how the oscillation process runs and how this process behaves under various parameters of the acceleration. The main parameters which impacts the process were found to be the laser beam intensity, the coaxial constant magnetic field intensity and the laser radiation wavelength. It has been shown the way of achieving the sufficiently high energy particles without the application of a very high magnetic.

2. Equations for charged particle trajectories and kinetic energy

In this section we derive analytic trajectories for a free electron moving in a plane-wave laser field and an externally applied strong uniform magnetic field aligned in the direction of propagation of the laser field. The electron dynamics resulting from interaction with these fields we have analyzed in the case of a circular polarized electromagnetic monochromatic wave in the lossless conditions [46]. Dynamical relativistic equation and the continuous equation of normalized energy γ in this case have the following form

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\left[\vec{V}\times\left(\vec{B}+\vec{B}_z\right)\right],\qquad \frac{d\gamma}{dt} = \frac{q}{m_0c^2}\vec{V}\cdot\vec{E} \tag{1}$$

where p, q and m_0 are the momentum, charge and the rest mass of the particle, E and B are the electric and magnetic field strengths of the electromagnetic wave, c is the velocity of the electromagnetic wave, V is the particle velocity and B_z is the external static magnetic induction in direction along the z coordinate
and

$$\gamma = \left(1-\beta^2\right)^{-\frac{1}{2}},\quad \vec{\beta} = \frac{\vec{V}}{c},\quad \vec{p} = \gamma\cdot m_0c\vec{\beta},\qquad \beta^2 = \beta_x^2+\beta_y^2+\beta_z^2,\quad \beta_{x,y,z} = \frac{V_{x,y,z}}{c} \tag{2}$$

where $\beta = V/c$.

The solving procedure of these equations are presented in our paper [47]. As a result we obtained uncomplicated expressions for trajectory coordinates in the form of

$$x = \eta\sin\phi + (\eta-\xi)\sin\chi\phi,\qquad y = \eta\cos\phi - (\eta-\xi)\cos\chi\phi - \xi, \tag{3}$$

$$z = \frac{c}{2\omega\gamma_0^2\left(1-\beta_{0z}\right)^2}\left\{\left[1+2\vartheta^2-\gamma_0^2\left(1-\beta_{0z}\right)^2\right]\phi - \frac{2\vartheta^2}{1+\chi}\sin(1+\chi)\phi\right\} \tag{4}$$

where $\eta, \chi, \xi, \vartheta$ are constant parameters defined as

$$\eta = \frac{\chi\xi}{1+\chi},\quad \chi = \frac{B_z}{\gamma_0\left(\beta_{0z}-1\right)B_c},\qquad \xi = \frac{cB_0}{B_z\omega},\quad \vartheta = \frac{-B_0}{(1+\chi)B_c} \tag{5}$$

where the phase of the field ϕ stands for combination ωt - kz and has a form

$$\phi = \omega\left[t-\frac{z(t)}{c}\right], \tag{6}$$

Equations (3,4) completely describe the trajectory of the accelerated charged particle as a result of acceleration by a laser or maser beam and additionally axially applied a static magnetic field. The equations are valid also in the relativistic region. In order to find the kinetic energy of the particle in the relativistic region the variation in time the velocity of it should be determined. This can be achieved using the derived by the authors of this paper the relations for the components of a particle velocity in the form:

$$V_x = \frac{c\vartheta}{\gamma}\left(\cos\phi-\cos\chi\phi\right),\qquad V_y = -\frac{c\vartheta}{\gamma}\left(\sin\phi+\sin\chi\phi\right),\qquad V_z = c\frac{1+2\vartheta^2\left[1-\cos(1+\chi)\phi\right]}{2\gamma_0\gamma\left(1-\beta_{0z}\right)} - \frac{1}{2\gamma}c\gamma_0\left(1-\beta_{0z}\right) \tag{7}$$

Knowledge of this components enables to determine the relativistic kinetic energy

$$E_k = m_0 c^2 (\gamma - 1) \tag{8}$$

3. Physics of the acceleration process

The action of combined electric and two magnetic fields is rather complicated to describe. In order to clear the problem let us start with the case of linear polarized laser beam. An electron will be accelerated from the rest by the electric component of the laser field and will start moving in the negative *x* direction following the turn-on of the laser. However, from this moment on the electron will start to act the bending effects of the magnetic component *B* of the laser field and static magnetic field B_z will start to influence the electron motion. The component *B* will tend to bend the electron trajectory toward the axis direction while the component B_z will tend to make the electron follow a helix around the *z* axis. The result of these combined actions is the helical trajectory with superimposed wiggles.

Now let us look at the case of circular polarization. Now the electric field $\vec{E}$ of the laser radiation rotates around the *z* axis. As a result of this acceleration of the particle occurs. The particle will start moving along the direction of the circularly rotating $\vec{E}$ vector. However, from this moment on the particle will start to act the bending effect connected with the Lorentz force by the magnetic component *B* of the laser field. The component *B* will start to bend the particle trajectory in the co-axial direction (the z axis). This will create the V_z component of the particle's velocity. While the external static magnetic field B_z will tend to make the electron follow a helix around the *z* axis. The particle will move along the helix winding around the *z* axis with rising the radius. The result of these combined actions is the helical trajectory with superimposed wiggles [10,29]. The wiggles disappear at the resonance conditions. This condition is connected with the effective energy transfer from the electromagnetic field to the particle. The transfer appears when B_z approaches the value of the cyclotron magnetic induction $B_c = m_0\omega/q$. At this condition the maximal absorption by the particle occurs. The resonance value of B_z does not depend of the electric field intensity but depends on the wavelength, a mass and a charge of the particle.

During acceleration in the laser beam the energy transfer rate from the electromagnetic field to the particle can be found from the energy equation of conservation [48]

$$\frac{d\gamma}{dt} = \frac{q}{m_0 c}\vec{\beta}\cdot\vec{E}$$

This equation describes the rate at which the particles energy changes with time. This rate is proportional to $\vec{\beta}\cdot\vec{E}$ where $\vec{\beta}c$ is the particle's velocity vector, and $\vec{E}$ is the electric field of radiation. The equation shows that the rate at which the energy transferred to the particle will be maximized if the vectors $\vec{\beta}$ and $\vec{E}$ are parallel, and retain their orientation over many oscillations. In this magnetic field a particle's velocity vector and the electric field vector rotate in the same direction if the radiation is circularly polarized. $\vec{E}$ oscillates so rapidly that $|\vec{\beta}\cdot\vec{E}|$ nearly averages to zero over any macroscopic time scale. The key to achieving large energy transfers is to make $\vec{\beta}$ rotate rapidly (with $\vec{E}$) for long time. If the radiation is circularly polarized, it is possible to choose initial parameters such that $\vec{\beta}$ and $\vec{E}$ maintain their relative orientation through the large time of the vector $\vec{E}$ oscillation period and allows significant energy transfer to occur. For most choices of parameters, the energy is an oscillatory functions of time. The particle alternately gains and losses energy. However, if the parameters are such that $\vec{\beta}$ and $\vec{E}$ rotate synchronously, the particle's energy is not a periodic function but increases or decreases monotonically. This can be achieved by application of the static magnetic field of the value near to the resonance condition, now the energy no longer oscillates, it keeps rising in a monotonic way. However, the required intensity of the magnetic field was found to be very large.

In the resonance region efficient coupling between the particle and radiation field can occur and the interaction is resonant. Generally, such a synchronization can be achieved by combination of the particle initial velocity, static longitudinal magnetic induction and the laser radiation frequency. The sign depending on the initial orientation of $\vec{\beta}$ and $\vec{E}$ ie on the angle $\theta(t)$ between the rotating electric field vector and the rotating perpendicular to the axis component of the particle's velocity vector

$$\vec{\beta}_\perp \cdot \vec{E} = \beta_\perp E\cos\theta$$

As the orientation of these vectors changes in time, the energy of the particle must also change. In such a case we will observed a periodic function of particle's kinetic energy in time. Only in the resonance case, when $\theta = 0$, ($\cos\theta = 1$), or $\theta = \pi$ for electron, ie as the rate of rotation of $\vec{\beta}$ approaches to that $\vec{E}$, the particle can get the maximum energy from the radiation electric field and efficient energy transfer is possible. If B_z is chosen such that the difference in the particle rotation and the electric amplitude vector *E* frequencies

$$\Delta\omega = \frac{qB_z}{\gamma_0 mc} - (1 - \beta_{z0})\omega = 0$$

(the resonance condition), θ does not oscillate, but (for electron) evolves monotonically toward π. So if a homogeneous, monoenergetic beam of electrons is injected into the interaction region with $\Delta\omega = 0$, the energy change of a particular electron, depends on its initial direction with respect to the $\vec{E}$ vector of the laser radiation: those with θ_0 between π/2 and 3π/2 are accelerated, the others are decelerated. If $\theta_0 = \pi$ and

$\Delta\omega = 0$, than $\theta(z)$ is constant, and $\vec{\beta}$ and $\vec{E}$ rotate at exactly the same rate. In this case the electron acceleration is maintained. Therefore, if two particles are injected with the same energy but different angles, the particle with the smaller angle θ_0 will be accelerate longer and to higher energy. So if the charged particle is injected into the resonance interaction region with velocity $\vec{V}_{\perp}$, the energy of the particle depends on the their initial velocity direction in the *(x, y)* plane.

4. Acceleration process with particle initially at rest

It follows from (7) that the total velocity of the particle accelerated by the laser or maser radiation has the form:

$$V^2 = V_x^2 + V_y^2 + V_z^2 = 2\frac{c^2\vartheta^2}{\gamma^2}\left[1-\cos(1+\chi)\phi\right] + \\ + \frac{c^2\vartheta^4}{\gamma^2}\left[1-\cos(1+\chi)\phi\right]^2$$

Substitution for γ its relation (2), gives the relation

$$V = \pm c\vartheta\sqrt{\frac{A}{1+\vartheta^2 A}} = \pm c\sqrt{\frac{\vartheta^2 A}{1+\vartheta^2 A}} \tag{9}$$

where

$$A = 2\left[1-\cos(1+\chi)\phi\right] + \vartheta^2\left[1-\cos(1+\chi)\phi\right]^2 \tag{10}$$

Using (9) we get the relativistic expression for normalized energy:

$$\gamma = \frac{1}{\sqrt{1-V^2/c^2}} = \frac{1}{\sqrt{1-V_c^2/c^2}} = \\ + \frac{1}{\sqrt{1-\frac{\vartheta^2 A}{1+\vartheta^2 A}}} = \sqrt{1+\vartheta^2 A} \tag{11}$$

In the conditions beyond the resonance the velocity as well as kinetic energy show the oscillatory behavior. Now we will found the conditions at which the extreme values of the energy appear. Equation (11) is the subject of our analysis. For example, Figs. 1a, b, c show the energy evolution in time under assumed acceleration parameters concerned the laser and the constant coaxial magnetic field. When magnetic field is not within the resonance region, the oscillations of the energy gained by a particle occur (Figs. 1a and 1b). From Eqn (11) it follows that the function $\gamma(\phi)$ exhibits the maxima and the minima at certain moments. The constant coaxial magnetic field B_z can influence on the moments these extremes appear. Now we will analytically find the moment of appearance of the above mentioned extremes of the $\gamma(\phi)$ function.

Since

$$\frac{d\gamma}{d\phi} = \frac{d}{d\phi}\sqrt{1+\vartheta^2 A} = \frac{1}{2}\vartheta^2\left(1+\vartheta^2 A\right)^{-\frac{1}{2}}\frac{dA}{d\phi} = \\ = \frac{\vartheta^2}{2\gamma}\frac{dA}{d\phi}$$

The extremes can be found from the condition:

$$\frac{d\gamma}{d\phi} \equiv \frac{dA}{d\phi} = 0$$

From (10) we get

$$\left[1+\vartheta^2 - \vartheta^2\cos(1+\chi)\phi\right]\sin(1+\chi)\phi = 0$$

Since

$$1+\vartheta^2 - \vartheta^2\cos(1+\chi)\phi \neq 0$$

the extremes can be found from relation

$$\sin(1+\chi)\phi = 0$$

and they appear in the moments for which

$$\phi_n = \pm\frac{n\pi}{1+\chi} \tag{12}$$

The condition (12) indicates that the following equations are valid:

$$A_{\min} = 0, \quad for \cdot n = 0,2,4,.... \\ A_{\max} = 4\left(1+\vartheta^2\right) \quad for \cdot n = 1,3,5,.... \tag{13}$$

$$\gamma_{\min} = 1 \\ \gamma_{\max} = \sqrt{1+4\vartheta^2\left(1+\vartheta^2\right)} = 1+2\vartheta^2 \tag{14}$$

The presented analysis shows that the kinetic energy of the particle oscillates as a function of the laser radiation phase ϕ approaching the maximal values of the equal levels corresponding to the energy γ_{max} (Figs.1a and 1b). From the presented Figures as well as from Eqn (14) it follows that the level of the maximal energy increases with the induction B_z of the constant magnetic field. When $B_z \to B_c$, $\gamma_{max} \to \infty$ [30].

From Eqn (12) can be seen that the phase interval between the subsequent maximum of $\gamma(\phi)$ is expressed by the following relation

$$\Delta\phi = \phi_n - \phi_{n-1} = \frac{\pi}{1+\chi} = \frac{\pi}{1-B_z/B_c} \tag{15}$$

From Eqn (15) it follows that when the B_z magnetic field increases, the phase interval between the subsequent maxima rises. It is easy to show that time interval between the maxima is

$$\Delta t = \frac{\pi}{(1+\chi)\omega} + \frac{2\pi E_0^2 B_c}{\omega c^2\left(B_c - B_z\right)^3} \tag{15a}$$

a)
b)
c)

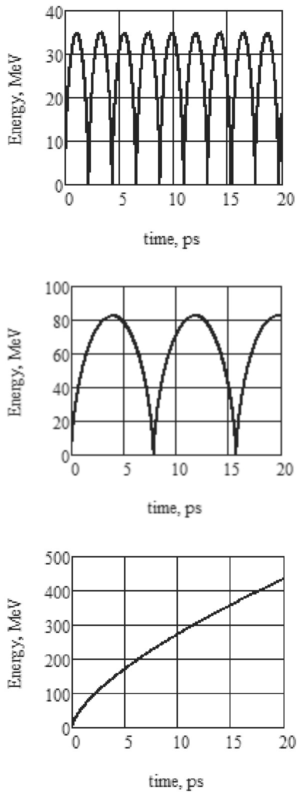

Fig. 1. Changes in the kinetic energy of an electron accelerated in the laser beam and a constant axial magnetic field. Parameters: $\lambda = 10\ \mu m$, $E_0 = 10^{12}\ V/m$, a) $B_z = -500\ T$, b) $B_z = -700\ T$ and c) $B_z = -1071\ T$.

A simple calculation shows that the results obtained from Eqn (15a) are in coincidence with that shown in Figs. 1 and 2 which are drawn using the analytical Eqn (8). From the Figures (compare Fig. 1b with Fig. 2a) and from above equation also can be seen that the time intervals between the maxima rises with the laser intensity of the electric field E_0 at the same B_z. It is clearly shown in Figs. 1a and 1b, where instead of normalized energy function, the evolution in time of the electron kinetic energy is presented. When B_z enters into the resonance region which means that $B_z \to B_c$, the above mentioned phase intervals rise very rapidly which means that $\Delta t, \Delta\phi \to \infty$ or and eventually the process enters the resonance region (Fig. 1c). The maximal kinetic energy can be obtained combining Eqns (8) and (14)

$$E_k^{\max} = m_0 c^2 \left(\gamma_{\max} - 1\right) = 2 m_0 c^2 \vartheta^2 \tag{16}$$

In the resonance conditions when $B_z \to B_c$, the parameter

$$\vartheta = \frac{B_0}{B_z - B_c} \tag{17}$$

tends to be very large, $\vartheta \to \infty$ and it means that $E_k^{\max} \to \infty$. For $B_z \to B_c$ the parameter $\vartheta \to \infty$ and it is easy to show that

$$\frac{V_c^{\max}}{c} \to 1 \text{ and } V_c \to c.$$

The magnetic field amplitude depends on the laser beam intensity and it means that B_0 varies along with E_0. From Eqn (16) can be seen that the maximal value of the kinetic energy depends on the laser beam intensity. The calculation results show it clearly, the maximal value of the oscillating energy rises with the beam intensity with the same constant magnetic field (compare Fig. 1b with Fig. 2a).

a)
b)

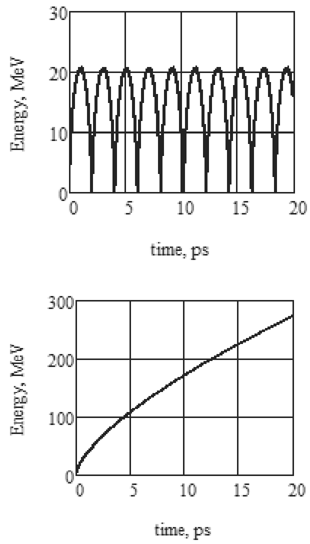

Fig. 2. Changes in the kinetic energy of an electron accelerated in the laser beam and a constant axial magnetic field. Parameters: $\lambda = 10\ \mu m$, $E_0 = 5\text{x}10^{11}\ V/m$, a) $B_z = -700\ T$ and b) $B_z = -1071\ T$ (the resonance condition).

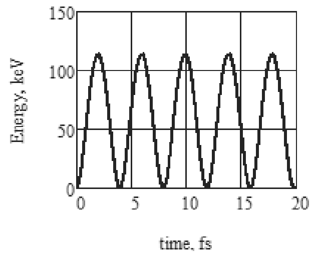

Fig. 3. Changes in the kinetic energy of an electron accelerated in the laser beam and a constant axial magnetic field. Parameters: $\lambda = 1\ \mu m$, $E_0 = 10^{12}\ V/m$, $B_z = -700\ T$

The wavelength also impacts the course of the acceleration process. Fig. 3 shows that with the decrease of the laser radiation wavelength, at the maintained B_z and E_0, the maximal value of the energy decreases and the time interval between the maximal values also decreases. This can be supported by Eqns (15a).

The distance the particle penetrates (Eqn (4)) during its acceleration initially at rest, can be simplified to the following form

$$z = \frac{c\vartheta^2}{\omega}\left[\phi - \frac{1}{1+\chi}\sin(1+\chi)\phi\right]$$

It is possible to search for the extremes as it has been done for the energy.

$$\frac{dz}{d\phi} = \frac{c\vartheta^2}{\omega}\left[1-\cos(1+\chi)\phi\right] = 0$$

This means that

$$\cos(1+\chi)\phi = 1$$

and extremes appear at the moments defined by the following condition

$$\phi_n = \pm\frac{2\pi n}{1+\chi} \tag{18}$$

The second derivative

$$\frac{d^2z}{d\phi^2} = \frac{c\vartheta^2}{\omega}\sin(1+\chi)\phi$$

substituting of Eqn (18) gives

$$\frac{d^2z}{d\phi^2} = 0$$

From this result we can state that at the moments defined by Eqn (18) there are no extremes but the narrow intervals during which the distance z does not increase nor decrease. This is clearly shown in Fig. 4. Which indicates that the particles velocity component $V_z = 0$ at these moments. The $z(t)$ curve at these moments appear to be flat - horizontal. The positions under question can be calculated from relation

$$z_n^{flat} = \frac{c\vartheta^2}{\omega}\phi_n = \frac{c\vartheta^2}{\omega}\frac{2\pi n}{1+\chi} = \frac{cB_0^2 B_c}{\omega(B_c - B_z)^3}2\pi n$$

The spatial interval between the subsequent flat points z^{flat}

$$\Delta z^{flat} = z_n^{flat} - z_{n-1}^{flat} = \frac{2\pi c B_0^2 B_c}{\omega(B_c - B_z)^3}$$

$$\Delta\phi = \phi_n - \phi_{n-1} = \frac{2\pi}{1+\chi}$$

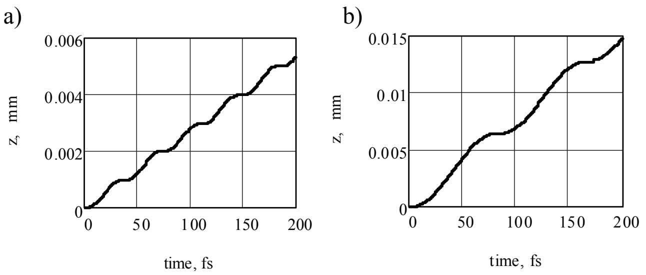

Fig. 4. Changes in the z component of the electron trajectory during acceleration process in the laser beam and a constant axial magnetic field. Parameters: $\lambda = 10$ µm, $E_0 = 10^{11}$ V/m, a) $B_z = -10$ T, b) $B_z = -500$ T.

which shows that when the constant magnetic field rises the distance between the flat regions also rises (compare Fig. 4a with 4b). The corresponding phase differences

In the case when $B_z \to B_c$, $z_n^{flat} \to \infty$. It is the resonance condition and $\Delta\phi$ begins to be very large. Which shows that within the resonance conditions there are no the flat intervals at all. The velocity of the particle continuously rises and the moments where velocity $V = 0$ no longer exist.

5. Acceleration process of particle initially moving

Let us assume that the initial velocity is not zero e.i.

$$\gamma_0 \neq 1,\ \ \beta_{0z} \neq 0$$

where

$$\gamma_0 = \frac{1}{\sqrt{1-\beta_{0z}}}$$

Then we can obtain the following relation

$$\gamma_0(1-\beta_{0z}) = \sqrt{\frac{c-V_{0z}}{c+V_{0z}}}$$

The axial magnetic field $B_z = m_0\omega_c/q$, where ω_c is the cyclotron angular frequency of the charged particle moving in the constant magnetic field of induction B_z. From (5) we obtain

$$\chi = -\frac{B_z}{\gamma_0(1-\beta_{0z})B_c} = -\frac{\omega_c}{\omega}\sqrt{\frac{c+V_{0z}}{c-V_{0z}}}$$

The extremes of the $\gamma(\phi)$ function at nonzero initial velocity of the particle, $\beta_{0z} \neq 0$ can be obtained from the relation

$$\frac{d\gamma}{d\phi} = 0$$

The normalized energy has the form:

$$\gamma = \frac{1+2\vartheta^2\left[1-\cos(1+\chi)\phi\right]}{2\gamma_0(1-\beta_{0z})} + \\ + \frac{1}{2}\gamma_0(1-\beta_{0z}) \tag{19}$$

hence

$$\frac{d\gamma}{d\phi} = \frac{\vartheta^2(1+\chi)}{\gamma_0(1-\beta_{0z})}\sin(1+\chi)\phi = 0$$

The extremes can be found from

$$\sin(1+\chi)\phi = 0$$

and they appear at the moments in which

$$\phi_n = \pm\frac{n\pi}{1+\chi}$$

Thus from (19) we can obtain the extremes of the $\gamma(\phi)$ function

$$\begin{aligned} &for\ n=0,2,4,... \\ &\gamma_{\min}=\frac{1}{2\gamma_0(1-\beta_{0z})}+\frac{1}{2}\gamma_0(1-\beta_{0z}), \qquad a) \\ &for\ n=1,3,5,... \\ &\gamma_{\max}=\frac{1+4\vartheta^2}{2\gamma_0(1-\beta_{0z})}+\frac{1}{2}\gamma_0(1-\beta_{0z}) \qquad b) \end{aligned} \tag{20}$$

Considering that for non zero initial velocity of the particle the ϑ parameter is

$$\vartheta=\frac{B_0}{\dfrac{B_z}{\gamma_0(1-\beta_{0z})}-B_c}$$

(20b) relation can be presented in the form

$$\gamma_{\max}=\frac{2B_0^2}{[B_z-\gamma_0(1-\beta_{0z})B_c]^2}+\frac{1}{2\gamma_0(1-\beta_{0z})}+ \\ +\frac{1}{2}\gamma_0(1-\beta_{0z}) \quad for\ n=1,3,5,...$$

when $B_z\to\gamma_0(1-\beta_{0z})B_c$, the maximal energy $\gamma_{\max}\to\infty$. When initial velocity equals zero the above presented results get the form of Eqns (14).

6. Conclusions

The resonance is the most important region of the acceleration process of the charged particles by the laser beam. The conditions to establish the resonance acceleration are difficult to achieve because very high constant magnetic field is needed, especially in the laser case. As it is well known from many publications and the obtained in this paper results outside the resonance conditions the oscillation of the kinetic energy of the particle occurs under action of the axial magnetic field which is significantly reduced. At the authors best knowledge the research results on the dependence of the maximal value of the energy and the frequency of its oscillations on different parameters have not been published yet. We have studied this range of acceleration analytically and the results are illustrated. We found this region of the acceleration process interesting since pulses of energetic particles of lower energy than in the resonance conditions may be used in many applications in which the maximal energy and the frequency of its appearance may be important. The presented results indicate that the kinetic energy periodically achieves the maximal energy the value of which rises with the increasing magnetic field B_z and the laser beam intensities and finally when $B_z\to B_c$, the energy $\gamma_{max}\to\infty$. The time interval of the moments at which subsequent maxima occur increases with the constant magnetic field and laser beam intensities. From Eqn (12) it results that for initial velocity being zero as well as for nonzero initial velocity the phase interval between subsequent maximal energies can be find from the following equation:

$$\Delta\phi=\phi_n-\phi_{n-1}=\frac{\pi}{1+\chi}=\frac{\pi}{1-B_z/B_c}$$

This equation shows that when magnetic induction B_z rises, the interval between the maxima rises as well. When B_z approaches the resonance region i.e. when $B_z\to B_c$, the intervals begin to be very large. The maximal value of the oscillating energy and the time intervals between the maxima rise with the beam intensity at the maintained constant magnetic field B_z.

The wavelength also impacts the course of the acceleration process. The maximal value of the energy and the time interval between the maximal values decrease with the decrease of the laser radiation wavelength, at the maintained B_z and E_0.

The penetration distance of the accelerated particle along the axis rises monotonically only at the resonance conditions. Outside the resonance the flat intervals appear which indicate that at these moments the particle velocity is zero. This corresponds to the moments when the minima of the energy occur. The distance between the flat regions rises with the constant magnetic field intensity.

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