

Formulating of reverse task of chosen class of mechatronic systems

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Analysis and modelling

ABSTRACT

Purpose: of this paper is modelling by means of the first and second category graphs and analysis of vibrating subsystem of mechatronic systems by means of the exact and approximate methods.

Design/methodology/approach: Approach was to nominate the relevance or irrelevance between the characteristics obtained by means of the exact method (only for the mechanical subsystem) and the approximate method. Such formulation concerns mostly the relevance of the natural frequencies-poles of the characteristics both mechanical subsystems and mechatronic systems.

Findings: are approximate solutions requiring all the conditions for torsionally vibrating mechanical and/or mechatronic systems. It is an introduction to synthesis of these systems modelled by graphs of the considered category.

Research limitations/implications: is both torsional vibrating continuous mechanical subsystem and mechatronic systems of the linear continuous type.

Practical implications: of this work is to present the introduction to synthesis of considered class of mechatronic bar-systems with a constant changeable cross-section.

Originality/value: Originality of such formulation is focused on the use of the different category graphs for modelling and synthesising by means of the continued fraction expansion method represented by graphs of torsionally vibrating bars to the synthesis of discrete-continuous mechatronic systems.

Keywords: Applied mechanics; Exact and approximate methods; Graphs and hypergraphs; Vibrating subsystem of mechatronic system

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1. Introduction

In last years (comp. [17,19,21]), a lot of attention is paid to the researches connected with new construction solutions, especially as far as the technology of drives, which depend on the phenomenon of piezoelectricity and electrostriction. The piezoelectric elements are also used to eliminate the oscillation [18]. PZT plates including damping are presented in [25,26].

The problems of analysis of vibrating systems, discrete and discrete-continuous mechanical systems by means of the

structural numbers methods modelled by the graphs, hypergraphs or classical methods have been made in the Gliwice Research Centre (e.g.[1-9, 21-24]). The problems of synthesis of electrical systems [1] and of the selected class of continuous, discrete - continuous or discrete mechanical systems and active mechanical systems concerning the frequency spectrum have been made. The continuous-discrete torsionally and flexibly vibrating mechatronic systems were considered in e.g. [10-12,23,24]. The approximate method of analysis called Galerkin's method has been used to obtain the frequency-modal characteristics. For comparison of obtaining dynamical characteristics - dynamical flexibilities only

for mechanical torsionally vibrating bar and flexibly vibrating beam, as a part of complex mechatronic systems, exact method and Galerkins method were used for example in [10-16,22,23]. In this paper frequency - modal analysis, modelling and synthesis of the dynamical characteristic of mechanical subsystem of a mechatronic system by means of the continued fraction expansion method represented by graphs have been presented for the mechanical system. The models of the mechanical subsystem of mechatronic system were presented in the different category graphs. Such formulation can possibly be the introduction to synthesis of torsionally vibration of complex discrete - continuous mechatronic systems with a constant cross-section.

2. The exact and approximate methods obtaining of the dynamical flexibility of subsystem - torsionally vibrating bar - of a mechatronic system

The shaft with the constant cross section is considered, clamped on the left end and free on the right one. The shaft is made of material with a transverse modulus G and mass density ρ . The shaft inertia moment - J , and its angle dislocation $\varphi(x,t)$ at the time moment t of the lining shaft section within the distance x from the beginning of the system. Moreover the shaft is loaded by the harmonical moment $M(t)$ acting on the free end.

The equation of motion of the shaft equals:

$$c_{\varphi} \frac{\partial^2 \varphi}{\partial x^2} = J \frac{\partial^2 \varphi}{\partial t^2} \quad (1)$$

where: $c_{\varphi} = \frac{GI_0}{l}$ - torsional rigidity of the shaft, I_0 - polar inertia moment of the shaft cross section.

The boundary conditions on the shaft ends are the following:

$$\begin{cases} \varphi = 0 \text{ when } x = 0, \\ c_{\varphi} \frac{\partial \varphi}{\partial x} = M_0 \cos \omega t \text{ when } x = l. \end{cases} \quad (2)$$

Dislocation is the harmonic function because the excitation is the harmonic one that is as such:

$$\varphi(x,t) = X(x) \cos \omega t \quad (3)$$

Substituting (3) to (1) is obtained as:

$$c_{\varphi} \frac{\partial^2 X}{\partial x^2} + J \omega^2 X = 0 \quad (4)$$

Afterwards the general solution of expression (4) can be obtained in the following equation:

$$X(x) = A \cos \lambda x + B \sin \lambda x \quad (5)$$

where: A and B are any optional real constants and

$$\lambda l = \omega \sqrt{\frac{I}{c_{\varphi}}} \quad (6)$$

The solution (5) fulfills the boundary conditions for:

$$\begin{cases} A = 0, B = \frac{M_0}{c_{\varphi} \lambda \cos \lambda l} \end{cases} \quad (7)$$

It means that dislocation of the section x of the shaft is presented as:

$$\varphi(x,t) = \frac{\sin \lambda x}{c_{\varphi} \lambda \cos \lambda l} M_0 \cos \omega t \quad (8)$$

Basing on the expression (8) the dynamical flexibility is equal:

$$Y_{xl} = \frac{\sin \lambda x}{c_{\varphi} \lambda \cos \lambda l} \quad (9)$$

According to the approximate - Galerkin's method, the final solution will be searched within the sum of eigenfunctions which will respond to the variables of the time and dislocation, which are strictly accepted and fulfill the boundary conditions in form:

$$\begin{cases} \varphi(0,t) = 0, X(0)T(t) = 0, \\ c_{\varphi} \frac{\partial \varphi}{\partial x} \Big|_{x=l} = M, c_{\varphi} X'(l)T(t) = M. \end{cases} \quad (10)$$

where: $M = M_0 \cos \omega t$

Using the Galerkin's method the solution of the equation (1) is given in the following form:

$$\varphi(x,t) = \sum_{k=1}^{\infty} \varphi_k(x,t) = A_k \sum_{k=1}^{\infty} \sin \frac{(2k-1)\pi x}{2l} \cos \omega t \quad (11)$$

The solution of the differential equation (1) comes to fulfilling the appropriate derivatives of the above expression (11)

$$\begin{cases} \frac{\partial^2 \varphi_k(x,t)}{\partial t^2} = -A_k \omega^2 \sin \frac{(2k-1)\pi x}{2l} \cos \omega t, \\ \frac{\partial^2 \varphi_k(x,t)}{\partial x^2} = -A_k \left[\frac{(2k-1)\pi}{2l} \right]^2 \sin \frac{(2k-1)\pi x}{2l} \cos \omega t. \end{cases} \quad (12)$$

Substituting the derivatives (12) to the equation of motion is obtained

$$-c_\varphi A_k \left[(2k-1) \frac{\pi}{2l} \right]^2 \sin \frac{(2k-1)\pi x}{2l} \cos \omega t + J A_k \omega^2 \sin \frac{(2k-1)\pi x}{2l} \cos \omega t = M_0 \cos \omega t. \quad (13)$$

Making appropriate transformations, the amplitude value A_k of the vibrations takes form of

$$A_k = \frac{M_0}{J\omega^2 - c_\varphi \left[(2k-1) \frac{\pi}{2l} \right]^2}. \quad (14)$$

Using the equation (14) and substituting it to (10) the dynamical flexibility equals:

$$Y_{xl} = \sum_{k=1}^{\infty} Y_{xl}^{(k)} = \sum_{k=1}^{\infty} \frac{\sin \left[(2k-1) \frac{\pi}{2l} x \right]}{J\omega^2 - c_\varphi \left[(2k-1) \frac{\pi}{2l} \right]^2}. \quad (15)$$

The plots of expressions (9) and (15) have been presented for example in [10,15].

3. Graphs as supply formal nations to modelling of the considered systems

Definitions of graphs, as mathematical objects, have been presented on the basis of the literature. The bibliography of this subject is very extensive and regards the theory as well as its applications (see [1,2,5]).

Using the symbols introduced in papers [5,20], a following couple is called a *graph*

$$X = ({}_1X, {}_2X) \quad (16)$$

where: ${}_1X = \{1x_0, 1x_1, 1x_2, \dots, 1x_n\}$ -finite set of vertices, ${}_2X = \{2x_1, 2x_2, \dots, 2x_m\}$ -family of edges, being two-element subsets of vertices, in the form of $2x_k = (1x_i, 1x_j)$ ($i, j = 0, 1, \dots, n$).

Using notion of the graph and hypergraph and their connections with structural numbers [1] and the system of notation [5,20], methods of modification of transforming vibration systems as the task of the synthesis of dynamical characteristic - mobility have been presented.

A characteristic - dynamical flexibility is given in the form:

$$Y(s) = \frac{\sum_{i=0}^k c_i t h^i \Gamma s}{s \sum_{j=0}^l d_j t h^j \Gamma s}. \quad (17)$$

After transformations [4-6] is obtained:

$$V(s) = sY(s), \quad (18)$$

$$r = t h \Gamma s, \quad (19)$$

the mobility has been obtained as follows:

$$V(r) = \frac{\sum_{i=0}^k c_i r^i}{s \sum_{j=0}^l d_j r^j}. \quad (20)$$

where: $c_k, c_{k-1}, \dots, c_0, d_l, d_{l-1}, \dots, d_0$ are any optional real numbers, $\Gamma = \sqrt{\frac{\rho}{G}} L = \sqrt{\frac{\rho^{(i)}}{G^{(i)}}} L^{(i)}$, ρ -mass density, $G^{(i)}$ - Kirchhoff's modulus, $L = L^{(i)}$ - length of basic element, $s = j\omega$, $j = \sqrt{-1}$, $c_k, c_{k-1}, \dots, c_0, d_l, d_{l-1}, \dots, d_0$ -real numbers, i, j, k, l - natural numbers, $k, l = 1$.

This problem is an introduction to optimal control and will be discussed more in detail in this paper.

4. Modelling of torsionally vibrating subsystems of mechatronic systems by hypergraphs

In the modelling of the considered class of continuous systems, the dependence between the amplitudes of *generalized forces* ${}_2s_k \in {}_2S$ and *generalized displacements* ${}_1s_i \in {}_1S$ can be described by *dynamical flexibility* Y_{ik} [4,5] in a form of:

$${}_1s_i = Y_{ij} {}_2s_j, \quad (21)$$

where: ${}_2s_j = Q_j \sin \omega t = 1 e^{j\omega t}$, $j = \sqrt{-1}$, ω - frequency.

The use of a loaded graphs and hypergraphs (as models of torsionally vibrating mechanical and/or mechatronic systems (e.g. [3-15]) in such a way it can provide the basis for the formalization which is the necessary condition of numerical discretization of the considered class of continuous mechanical systems.

5. Algebraic representation of the model of the discussed class of mechanical and/or mechatronic systems by structural numbers

First category structural number $A = {}^1A$ of polar graph X_{00} is determined as a product (product of structural numbers) (comp. [1]) of n - one-row prime factors, created from designating the edges incidental to any of the $(n-1)$ vertices of polar graph X_{00} .

The determinant function of structural number $A - \frac{\det A}{U}$ or $\frac{\det A}{U}$ is described by:

$$\det_Z A = \sum_{j=1}^m \prod_{i=1}^n z_{a_{ij}} \tag{22}$$

$$\det_U A = \sum_{j=1}^m \prod_{i=1}^n u_{a_{ij}} \tag{23}$$

where: $z_{a_{ij}} \in Z$ - set of the polar equation coefficients of the elements of a multi-body vibrating system, also called dynamic rigidities, $u_{a_{ij}} \in U$ - set of inverse mobilities or mechanical impedencies (comp. e.g. [4,5]) of the elements of the mechanical system with converged parameters.

By means of the functions (22) and (23) the mobility of a multi-body system or its inversion (immobility) are designated as:

$$V_{a_r} = \frac{\det^k A_{a_r}}{\det^k A} \tag{24}$$

$$U_{a_r} = \frac{\det^k A_{a_r}}{\det^k A} \tag{25}$$

Also it is possible to designate the dynamic flexibility of the vibrating system with converged parameters:

$$Y_{a_r} = \frac{\det Z A_{a_r}}{\det Z A} \tag{26}$$

and its dynamic rigidity:

$$Z_{a_r} = \frac{1}{Y_{a_r}} = \frac{\det Z A}{\det Z A_{a_r}} \tag{27}$$

where: $V_{a_r}, U_{a_r}, Y_{a_r}, Z_{a_r}$ - mobility, inverse mobility, dynamic flexibility and dynamic rigidity designated along the chain a_r , $\det_U A_{a_r}, \det_U A_{a_r}$ - determinant functions of the algebraic derivative of structural number A along the chain a_r , $\det_U A, \det_U A$ - determinant functions of structural number A , designated on the sets of inverse mobilities U and dynamic rigidities respectively- which are the polar equation coefficients Z .

The equations presented in this part are the base of formulation of synthesis problems of considered class of vibrating systems which are discussed in the next sections of the paper.

6. The synthesis of the dynamical characteristic of mechanical subsystems of mechatronic systems by the continued fraction expansion method represented by graphs

In this paper the continued fraction expansion method was applied in order to synthesize the dynamical characteristic of the torsionally vibrating mechanical or/and mechatronic system with the cascade structure. This is the method of decomposition of characteristics into partial fractions presented by graphs.

The dynamical flexibility $Y(s)$ of the torsionally (or longitudinally [3-5]) vibrating mechanical continuous bar system is given in the following form:

$$Y(s) = \frac{c_k \text{th}^k \Gamma s + c_{k-1} \text{th}^{k-1} \Gamma s + \dots + c_0}{s(c_l \text{th}^l \Gamma s + c_{l-1} \text{th}^{l-1} \Gamma s + \dots + d_0)} \tag{28}$$

Using the transformation (18) the mobility $V(s)$ is obtained

$$V(s) = \frac{c_k \text{th}^k \Gamma s + c_{k-1} \text{th}^{k-1} \Gamma s + \dots + c_0}{d_l \text{th}^l \Gamma s + d_{l-1} \text{th}^{l-1} \Gamma s + \dots + d_0} \tag{29}$$

After the next transformation (19) called Richards' transformation [4,5], the mobility $V(r)$ is given in the form of:

$$V(r) = \frac{c_k r^k + c_{k-1} r^{k-1} + \dots + c_0}{d_l r^l + d_{l-1} r^{l-1} + \dots + d_0} \tag{30}$$

where: $k-l = 1$.

The method of the synthesis of transformed mobility function $V(r)$ is presented here, assuming the even number of elements, and when k is an even natural number, then $V(r)$ takes form:

$$V(r) = \frac{c_k r^k + c_{k-1} r^{k-1} + \dots + c_0}{d_{k-1} r^{k-1} + d_{k-3} r^{k-k} + \dots + d_1 r} \tag{31}$$

or

$$V(r) = \frac{L_k(r)}{M_{k-1}(r)} \quad (32)$$

Dividing the numerator by denominator in (32) the equation is obtained as the first step of the synthesis:

$$\begin{aligned} V(r) &= V_r^{(1)}(r) + \frac{L_{k-2}(r)}{M_{k-1}(r)} = V_r^{(1)}(r) + \frac{1}{\frac{M_{k-1}(r)}{L_{k-2}(r)}} = \\ &= V_r^{(1)}(r) + \frac{1}{U_2(r)} = \frac{r}{c_r^{(1)}} + \frac{1}{U_2(r)}, \end{aligned} \quad (33)$$

where: $c_r^{(1)}$ is value of "i" synthesized discrete elastic element.

Graphical representation of the implementation of equation (33) in a form of a graph is shown in Fig. 1.

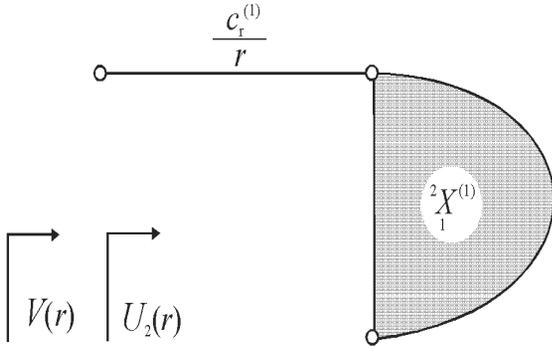


Fig. 1. Graphical illustration of equation (33)

The second step is the realization of the function $U_2(r)$ into (33). When dividing $M_{k-1}(r)$ by $L_{k-2}(r)$, $U_2(r)$ takes form:

$$\begin{aligned} U_2(r) &= U_z^{(2)}(r) + \frac{M_{k-3}(r)}{L_{k-2}(r)} = U_z^{(2)}(r) + \frac{1}{\frac{L_{k-2}(r)}{M_{k-3}(r)}} = \\ &= U_z^{(2)}(r) + \frac{1}{V_3(r)} = J_z^{(2)}r + \frac{1}{V_3(r)}, \end{aligned} \quad (34)$$

where: $J_z^{(i)}$ -value of "i" synthesized discrete inertial element.

The graph of synthesized mechanical bar system after operation (34) is shown in Fig. 2.

The synthesized mobility function after operations (33) is given in the following form:

$$V(r) = V_r^{(1)} + \frac{1}{U_z^{(2)}(r) + \frac{1}{V_3(r)}} \quad (35)$$

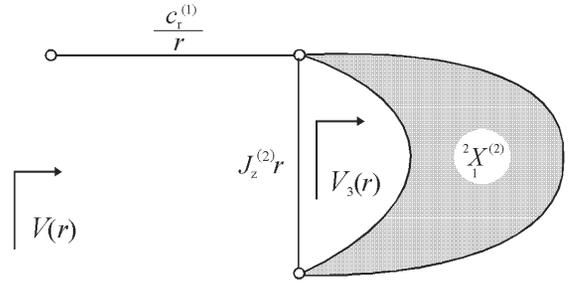


Fig. 2. Graphical illustration of equations (33) and (34)

The third step is the realization of the mobility function $V_3(r)$ into (35) is presented as

$$\begin{aligned} V_3(r) &= V_r^{(3)}(r) + \frac{L_{k-4}(r)}{M_{k-3}(r)} = V_r^{(3)}(r) + \frac{1}{\frac{M_{k-3}(r)}{L_{k-4}(r)}} = \\ &= V_r^{(3)}(r) + \frac{1}{U_4(r)} = \frac{r}{c_r^{(3)}} + \frac{1}{U_4(r)}. \end{aligned} \quad (36)$$

As a results of the equation (36) the mobility function $V(r)$ can be written as:

$$V(r) = V_r^{(1)} + \frac{1}{U_z^{(2)}(r) + \frac{1}{V_r^{(3)}(r) + \frac{1}{U_4(r)}}} \quad (37)$$

The equation (37) corresponds with the mobility function (31) of a graph (see Fig. 3). The mobility determined at the point indicated by the arrow is identical with (31). This graph is a model of discrete system but after transformation it is a continuous system (see [3]).

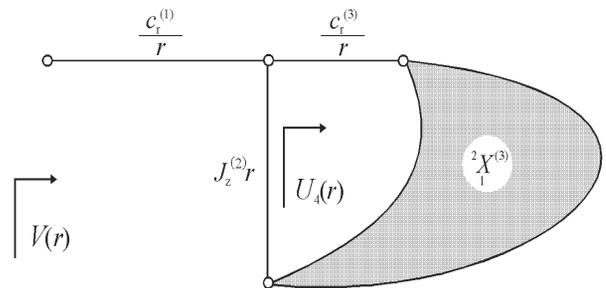


Fig. 3. Graphical illustration of the implementation of equation (37)

The process of the synthesis after steps (33-37) is to be continued until the function $U_k(r)$ is shown as:

$$U_k(r) = U_z^{(k)}(r) = J_z^{(k)}r \quad (38)$$

$$U(r) = J_z^{(1)} r + \frac{1}{\frac{r}{c_r^{(2)} + \frac{1}{J_z^{(3)} r + \frac{1}{\frac{r}{c_r^{(4)} + \dots} + \frac{1}{J_z^{(l-1)} r + \frac{1}{\frac{r}{c_r^{(l)}}}}}} \quad (44)$$

The Eq. (44) represents the inversion of mobility function of dynamical structure in the form of a polar graph X_{00} (Fig. 6).

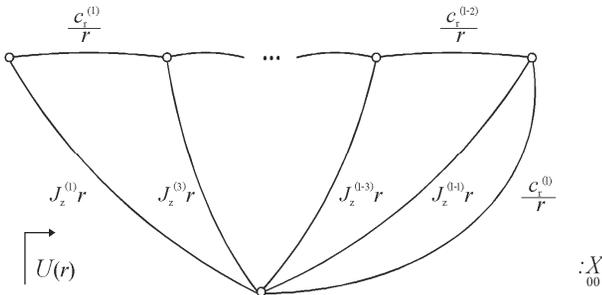


Fig. 6. Graphical illustration of equation (44)

The next case of dynamical characteristic - $U(r)$ of synthesized mechanical system is given, when k is an odd natural number and $k - l = -1$ and then $U(r)$ of the transformed function of bar system - is presented in the following form:

$$U(r) = J_z^{(1)} r + \frac{1}{\frac{r}{c_r^{(2)} + \frac{1}{J_z^{(3)} r + \frac{1}{\frac{r}{c_r^{(4)} + \dots} + \frac{1}{\frac{r}{c_r^{(l-1)} + \frac{1}{J_z^{(l)} r}}}}}} \quad (45)$$

The equation (45) represents the transformed inversion of the transformed mobility function of a dynamical structure in the form of a polar graph X_{00} (Fig. 7).

The above mentioned formulas and expressions from [5] are a base for the computer aided synthesis and structural modification of torsionally or longitudinally vibrating mechanical systems by means of the continued fraction expansion method.

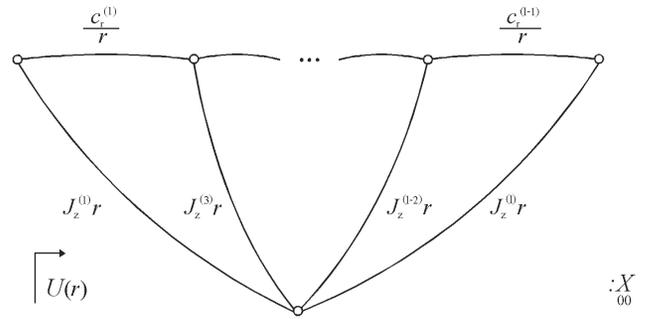


Fig. 7. Polar graph as an illustration of the implementation of the equation (45)

7. Last remarks

Applied methods and obtained results can be the introduction to the synthesis of torsionally vibrating mechatronic systems with a constant changeable cross-section. There are some numerical implementations [18,27-29] enabling to solve the considered systems. However, these problems shall be discussed in future research works.

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