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An estimation of the geothermal source power

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Analysis and modelling

ABSTRACT

Purpose: The purpose of the paper is to estimate the power of a geothermal source when the energy is obtained by means of a heat exchanger. It has been assumed that the temperature of the rocks at large distances from the heat exchanger is the same regardless of the direction and that the exchanger is sphere-shaped. Owing to this it is possible to estimate the power of the system without having to study the dynamics of the heat propagation in the rocks.
Design/methodology/approach: The applied method is to construct a mathematical model and its analysis. The system has been analysed in steady state.
Findings: The power of a geothermal source of energy depends, in steady state, on the radius of an exchanger in a linear way. Estimation the upper of the power that is attainable in a real system.
Research limitations/implications: Further investigation requires an analysis of transient processes the temperature in the rock during operation of geothermal energy.
Practical implications: The results presented in this paper can be used in the design of geothermal energy recovery systems.
Originality/value: The model and all the calculations are the author's results.
Keywords: Geothermal power plants; Renewable source of energy; Model; Steady state

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1. Introduction

It is expected that the source of the thermal energy inside the Earth is mainly different kinds of radioactive disintegration [1]. The current exploitation methods of the energy make use of the water of above 65°C that is in deep rock crevices. A closed circulation is forced where the water carries heat from the inside of the Earth up to the surface. The energy obtained may be used to warm buildings, produce electricity or in technological processes [2].

Making use of the water that is in direct contact with rocks is chiefly responsible for the failures in the geothermal heating systems as the rocks dissolve in water. That sort of water may show high chemical aggression. Besides, there occurs precipitation of minerals in the pipes, exchangers, valves and pumps of the system, which considerably increases the cost of its exploitation and is responsible for its breakdowns. That is why such systems have a limited energy efficiency since they can tap thermal energy only from relatively small depths at which water filled crevices can be found

An alternative, at present prospective source of renewable energy is the heat contained in waterproof rocks, mainly in granites that are found at a depth below 5000 m. The rock temperature at that depth may reach even 450°C. Owing to sealed heat exchangers placed at a great depth under the surface of the Earth it will be possible to acquire great amounts of energy via water fed from the surface which will have no direct contact with rocks. The water may circulate in a closed system and it may be treated before use. The nowadays applied technologies allow drilling down to 6000 m, with holes of a few meters in diameter that will enable the fitting in of effective heat exchangers.

The installations for acquiring energy at great depths will undoubtedly be expensive. Therefore, it seems advisable to make models of the whole system and then perform a simulation of their operation to minimize possible losses resulting from faulty installations.

2. Model and its analysis

Let us assume that an exchanger, closely fitted into the rocks, has been put at a great depth under the Earth. It is sphere-shaped, of radius R_1. The water flow is controlled in such a way that the rocks directly at the exchanger have a temperature of $T(R_1)$, constant in time. To make things simpler, let us assume that the temperature at a distance $R_2 >> R_1$ from the centre of the exchanger is the same in any direction of the exchanger and is $T(R_2) > T(R_1)$. It is these distant hot rocks that are the source of thermal energy. The situation is presented in Fig. 1.

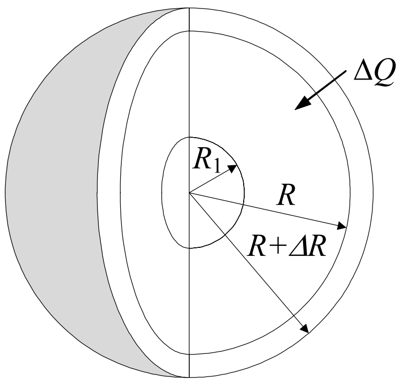

Fig. 1. Good quality figure with clear lettering

An operating heat exchanger cools the rocks in its environment. Heat reaches the exchanger through a thick layer of rocks. In steady state, the temperature of the rocks at any point is steady in time t. Thus, heat ΔQ flawing towards the exchanger through each sphere will be steady in time t. It follows from the law of conservation of energy that the flowing heat ΔQ will not depend on the radius R of the sphere. Consequently we have

$$\Delta Q(R,t) = const(R,t) = \Delta Q \tag{1}$$

The heat flowing through the sphere within the Δt time interval is

$$\Delta Q = \lambda \frac{S(R)\cdot[T(R+\Delta R)-T(R)]}{\Delta R}\cdot \Delta t\,, \tag{2}$$

where:

$S(R)$ - sphere surface of radius R,

$T(R)$ - temperature at distance R from the centre of the exchanger,

$\lambda\,[\frac{W\cdot m}{m^2\cdot {}^\circ C}]$ - thermal conductivity for rocks (rocks are assumed to be homogenous).

Then the power of the system is

$$M = \frac{\Delta Q}{\Delta t} = 4\pi\lambda\cdot R^2\frac{[T(R+\Delta R)-T(R)]}{\Delta R} = const(R,t) \tag{3}$$

If $\Delta t \to 0$, then, based on (3), we obtain

$$M = 4\pi\lambda\cdot R^2\frac{dT(R)}{dR}\,, \tag{4}$$

$$4\pi\lambda\cdot dT(R) = M\cdot\frac{1}{R^2}dR\,, \tag{5}$$

$$4\pi\lambda\cdot\int_{T(R_1)}^{T(R)} dT(R) = M\cdot\int_{R_1}^{R}\frac{1}{R^2}dR\,, \tag{6}$$

$$4\pi\lambda\cdot[T(R)-T(R_1)] = M\cdot\left[\frac{1}{R_1}-\frac{1}{R}\right]. \tag{7}$$

Finally, we arrive at

$$T(R) = \frac{M}{4\pi\lambda}\cdot\left[\frac{1}{R_1}-\frac{1}{R}\right]+T(R_1). \tag{8}$$

Within the $R_2 >> R_1$ distance from the centre of the exchanger the rock temperature is steady and is $T(R_2)$. Based on this boundary condition and equation (8) it is possible to determine the power of the system, M, after the substitutions of $T(R) = T(R_2)$ and $R = R_2$ in (8). Then, we obtain

$$M = \frac{4\pi\lambda\cdot[T(R_2)-T(R_1)]}{\frac{1}{R_1}-\frac{1}{R_2}}. \tag{9}$$

After putting (9) into (8) we obtain

$$T(R) = \frac{T(R_2)-T(R_1)}{\frac{1}{R_1}-\frac{1}{R_2}}\cdot\left[\frac{1}{R_1}-\frac{1}{R}\right]+T(R_1). \tag{10}$$

Thus, the temperature distribution for the rocks, $T(R)$, does not depend on the thermal conductivity for rocks λ. An example of the $T(R)$ temperature graph is shown in Fig. 2.

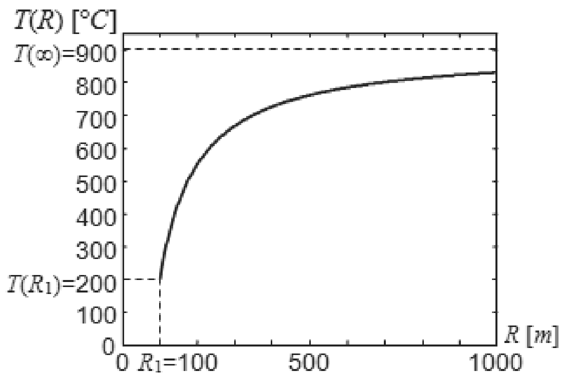

Fig. 2. Graph of $T(R)$ temperature function for R_1=100 m, T(R_1)=200°C, T(∞)=900°C

As $R_2 >> R_1$, then, based on (9), it can be assumed that

$$M \cong 4\pi\lambda \cdot [T(R_2) - T(R_1)] \cdot R_1 . \quad (11)$$

Hence, it follows that the power M of the system is proportional to the radius R_1 of the exchanger.

$$M \sim R_1 . \quad (12)$$

3. Etimation of the geothermal source power

Let us assume that the thermal conductivity of rocks (sandstone, quartzite, granite, anhydrite) is

$$\lambda \cong 3 \frac{W \cdot m}{m^2 \cdot °C} , \quad (13)$$

the temperature of the rocks at a large distance from the exchanger is

$$T(R_2) = 900°C , \quad (14)$$

and the temperature of the rocks at the exchanger is stabilized to

$$T(R_1) = 200°C . \quad (15)$$

Then, based on (11), the power of the system is

$$M \cong 12\pi[900 - 200] \cdot R_1 = 26376 \cdot R_1 \ [W] \quad (16)$$

The power of the system for the adopted values is shown in Fig. 3.

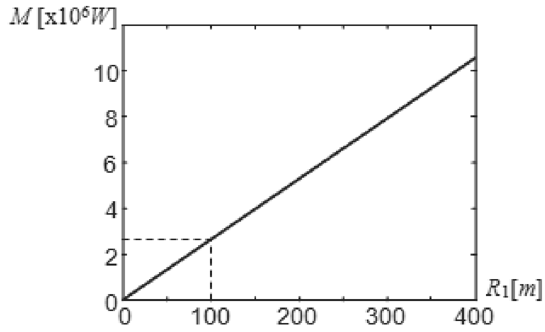

Fig. 3. Dependence of the powr of the system on radius R_1

Thus, in order to obtain 2,6 *MW*, it is necessary to use an exchanger with at least a 100 m radius.

4. Conclusions

The power of a geothermal source of energy depends, in steady state, on the radius of an exchanger in a linear way. That explains why it is essential to apply relatively large exchangers in order to acquire considerable powers.

A k-enlargement of the radius of the exchanger results in a k^2-enlargement of its surface. It might seem that the power of the system should be proportional to the exchanger surface. However, it follows from the analysis of the adopted model, that the power of the system is not proportional to the surface of the exchanger but to its radius. This is due to the effect of large rock layers which form a barrier for the inflowing heat.

The model under investigation assumes that the rock temperature is the same irrespective of the direction. In fact, high temperatures will occur in the rocks situated below the exchanger, from where heat energy will flow. The rocks located above the exchanger will not be a source of energy. Thus, the power of a real system will be lower than that estimated by means of the model under consideration. That is why the power presented by equation (11) must be treated as the upper estimation of the power that is attainable in a real system.

References

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[2] R.H. Kozłowski, Geothermic Power Plants of high capacity - how far?, Journal of Achievements in Materials and Manufacturing Engineering 49/2 (2011) 573-576.