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Adjustment method of parameters intended for first-principle models

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Analysis and modelling

ABSTRACT

Purpose: This paper demonstrates a process of estimation phenomenological parameters of a first-principle nonlinear model based on the hydraulic damper system.

Design/methodology/approach: First-principle (FP) models are formulated using a system of continuous ordinary differential equations capturing usually nonlinear relations among variables of the model. The considering model applies three categories of parameters: geometrical, physical and phenomenological. Geometrical and physical parameters are deduced from construction or operational documentation. The phenomenological parameters are the adjustable ones, which are estimated or adjusted based on their roughly known values, e.g. friction/damping coefficients.

Findings: A phenomenological parameter, friction coefficient, was successfully estimated based on the experimental data. The error between the model response and experimental data is not greater than 10%.

Research limitations/implications: Adjusting a model to data is, in most cases, a non-convex optimization problem and the criterion function may have several local minima. This is a case when multiple parameters are simultaneously estimated.

Practical implications: First-principle models are fundamental tools for understanding, optimizing, designing, and diagnosing technical systems since they are updatable using operational measurements.

Originality/value: First-principle models are frequently adjusted by trial-and-error, which can lead to non-optimal results. In order to avoid deficiencies of the trial-and-error approach, a formalized mathematical method using optimization techniques to minimize the error criterion, and find optimal values of tunable model parameters, was proposed and demonstrated in this work.

Keywords: First principle model; Data driven model; Hydraulic damper

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1. Introduction

According to the degree to which the a priori knowledge is available, then either a first-principle (FP) or a data-driven (DD) model, or a combination of both, can be applied [1-5]. First-principle models use understanding of the system underlying physics to derive its mathematical representation [6-8]. FP models are expensive in development since expertise in the area of knowledge at the advanced level is required to derive equations from physical laws, while data-driven models use system test data to derive its mathematical representation [9]. The advantage of the former approach is the depth of the insight into the behavior of the system and thus ability to predict the performance, while the advantage of the latter is the speed in which an accurate model can be constructed and confidence gained thanks to the use of the data obtained from the actual system. The difficulty of the former approach lies in the determination of the phenomenological parameters like the damping or the heat transfer coefficient. FP models are frequently adjusted by trial-and-error, which can lead to non-optimal results. On the other hand, the disadvantage of DD models is the need to handle multiple data sets in order to cover the range of system operation [10-11]. The goal is therefore to find a compromise and propose a combined first-principle data-driven model. Such models require a formal approach which allows the model parameters to be updated according to the operational data. In order to avoid the drawbacks of trial-and-error approaches, a formalized mathematical method using optimization techniques to minimize the error criterion is proposed for updatable first-principle models. It is believed that the smaller the number of updating model parameters, the more accurate the model and the faster the convergence of the algorithm used for model adjustment.

2. Experimental setup

Adjustment method of parameters intended for first-principle models is demonstrated based on the experimental setup and its theoretical model. A hydraulic damper system was proposed as a demonstrator since it involves multiple nonlinearities regarding state variables and parameters [11-13]. Therefore, a one-side hydraulic actuator equipped with a hydraulic accumulator was considered as the experimental setup. The accumulator allows to compensate a thermal expansion of the oil and the change in volume causes by the moving up-and-down piston-rod assembly. The considered hydraulic damper corresponds to a typical design of a monotube shock absorber commonly used in passengers or commercial vehicles [14]. The hydraulic damper uses a piston traveling within a single tube that is exposed more directly to the air facilitating cooling during high-speed or longer tests. To prevent foaming and bubbles in the oil, which degrades the force performance during longer tests, a gas-filled chamber of high-pressure gas is located in parallel to the oil chamber. This high-pressure gas makes it difficult for bubbles to form in the oil. A typical single-tube hydraulic damper is shown in Fig. 1.

The side of the piston attached to the rod is referred to as the rod side volume" or rebound volume and the side with the larger area is the head side volume or compression volume [14]. Oil occupies the tube volume on either side of the piston. The damper has a moving separator (floating piston) within the tube volume across from the head side of the piston. The additional piston separates the oil from a volume of gas under pressure (approximately 5-30 bar). During the compression stroke (rod moves inside the tube), the hydraulic fluid from the head side volume is forced through an arrangement of valves and orifices across the piston and into the rod-side volume. First the oil enters any of several port restrictions when pressure differential across a check valve exceeds a preset value. The fluid then enters a small junction volume within the piston before passing to the other side of the piston through a set of orifices referred to as the bleed leak restrictions. A second conduit opens from the junction volume to the other side of the piston through a pressure relief valve when the pressure differential exceeds a preset value. Oil can also leak around the gap between the piston seal- and the tube inner diameter. The relative incompressibility of oil and the fact that the displaced volume on the head side is larger than that of the rod side results in a reduction in the volume of gas to account for the additional volume of fluid on the head side which could not be forced to the rod side. During rebound stroke, the fluid on the rod side increases in pressure relative to the head side and oil flows across the piston to the head side through a separate set of ports and orifices than those active on the compression stroke. The compression ports are closed-off by a system of check valves during the rebound stroke and vice versa. As opposed to the compression stroke, however, the nitrogen gas volume provides compensation to decreasing oil volume.

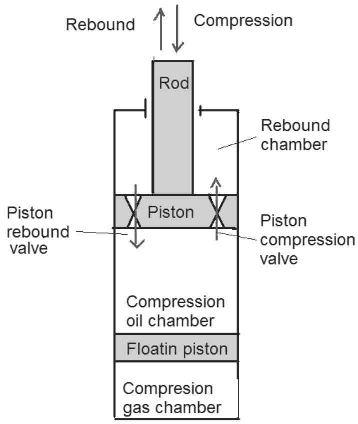

Fig. 1. Hydraulic damper scheme.

The hydraulic damper nonlinearities are related to a variable oil volume, friction of the main and floating piston, nonlinear valve characteristics, the gas and fluid model. The experimental setup (Fig. 2) of the damper allows to control:

- pressure of the gas in the hydraulic accumulator,
- mass ratio of the gas and hydraulic fluid by injection of small amount of gas directly to the rebound volume,
- valve characteristics defined by the disc springs and initial bleed areas

MTS 858 servo-hydraulic test rig (Fig. 3) was used to load a hydraulic damper and capture its dynamic characteristics, i.e. displacement vs. force. The MTS 858 test bench is capable of reproducing the shape of the desired sinusoidal signal under the load of the test dashpot up to the velocity of 0.7 m/s with sufficient accuracy. MTS company software was used to control the excitation and acquired measurement data. The damper setup is also equipped with the pressure sensors. Moreover the rod displacement and force are recorded. Data is sampled with at a frequency of 1024 Hz. The main components of the servo-hydraulic system are the hydraulic actuator with an integrated displacement transducer in a piston-rod assembly and the three-stage servo-valve system. The test rig is equipped with a PID-FF controller. The feed-forward (FF) section in this controller passes a proportion of the command signal to the controller output through a high-pass filter to block the command mean level. Different control settings are used depending on the type of excitation signal. The excitation signal is converted into a voltage applied to the servo-valve, which controls the amount of oil supplied to the chambers of the actuator.

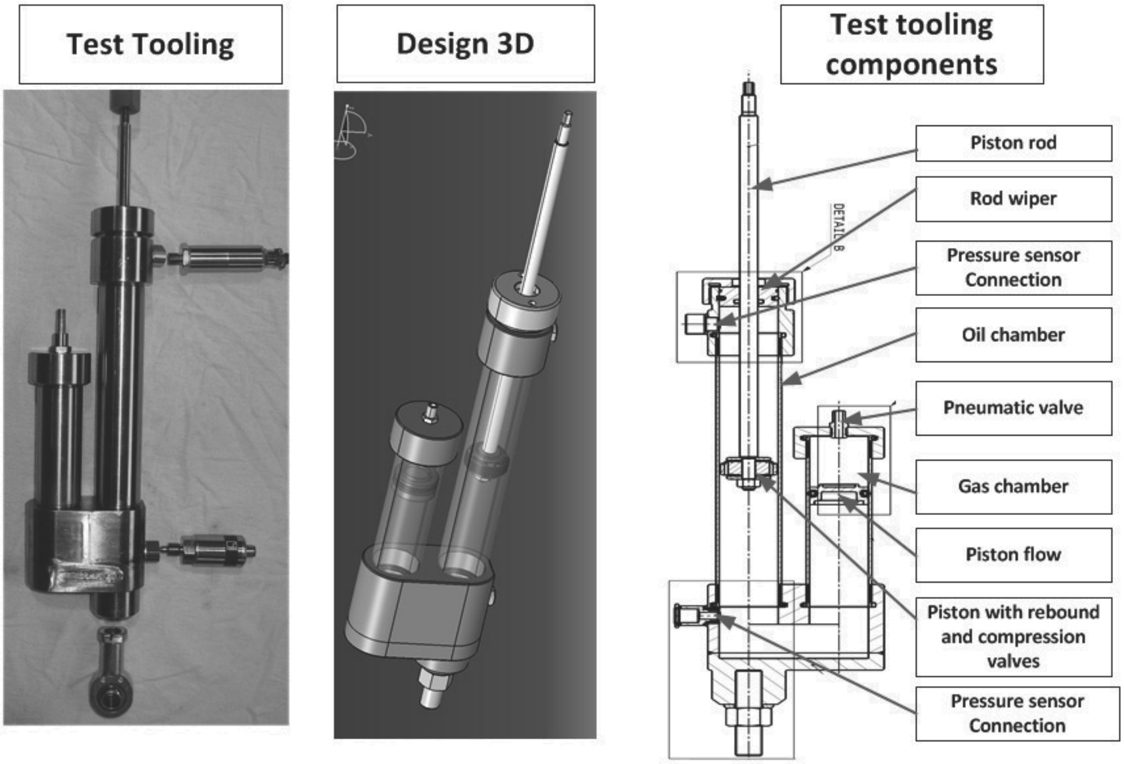

Fig. 2. Hydraulic test experimental setup

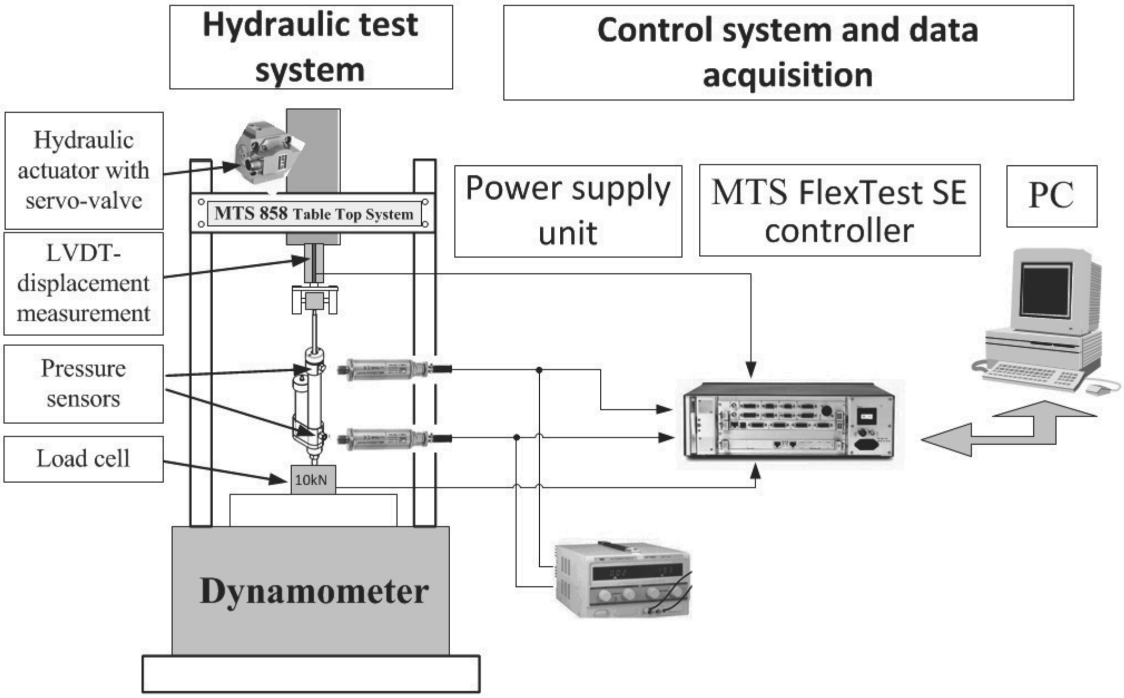

Fig. 3. Hydraulic damper and other components of the experimental setup

3. First-principle model formulation

Process of formulation of a first-principle data-driven model consists of three phases, namely (i) formulation of mathematical representation of first-principle laws, (ii) model adjustment and calibration process, and (iii) model evaluation phase.

The presented hydraulic damper model has been developed based on the following assumptions: (i) the dependency between density and pressure is non-linear (oil-gas emulsion), (ii) pressure and density are uniformly distributed in particular chambers, (iii) pressure-flow characteristics of all restrictions are given as monotonic functions, (iv) valves open and close abruptly in a completely symmetrical manner (valve dynamic is not considered), (v) oil temperature is constant, (vi) friction between floating piston and pressure tube is neglected (small compared to other frictions because of low friction sealing and lack of side force), and the mass of floating piston is neglected because is few times smaller than oil mass that inertia is also neglected.

3.1. Force equilibrium

Behavior of mono tube shock absorber connected witch top-mount can be described by following equation:

$$m_1 \cdot \ddot{x}_{rod} + c_{TM} \cdot (\dot{x}_{rod} - \dot{x}_{TM}) + k_{TM} \cdot (x_{rod} - x_{TM}) = F_d \tag{1}$$

where:

$m_1 = m_{rod} + 30\% \cdot m_{TM}$ is the mass of rod assembly (rod, piston, nuts, etc.) and mass of top-mount part that fixed to and displaces together with rod [kg];

c_{TM} - the top-mount damping coefficient [N·s/m];

k_{TM} - the top-mount stiffness [N/m];

x_{rod} - the rod displacement [m];

x_{TM} - the top-mount displacement (top mount part fixed to car or test machine) [m]

F_d - the force generated by a shock absorber [N].

The force F_d generated by a hydraulic damper is obtained by taking the equilibrium of forces acting on the inertial piston-rod assembly into account:

$$F_d = p_{reb} \cdot A_{reb} + p_0 \cdot A_{rod} - p_{com} \cdot A_{com} + F_{fric} \tag{2}$$

where:

A_{rod}, A_{com}, A_{reb} - the areas [m^2] of the rod, compression and rebound side of the piston;

p_{com}, p_{reb} - the pressures [Pa] in the compression and rebound chambers;

p_0 - the atmospheric pressure p_0=1e5[Pa];

F_{fric} - sum of dry friction force between the piston and the pressure tube and between rod and rod guide.

The sum of dry friction force F_{fric} between the piston and pressure tube and between rod and rod guide is modelled as follows:

$$F_{fric} = F_{fric\_max} \cdot \tanh\left(\frac{\dot{x}_{tube} - \dot{x}_{rod}}{v_{ref}}\right) \tag{3}$$

The friction force F_{fric} depends on the direction of relative rod travel not on velocity nor piston position. The maximal friction force F_{fric_max} is obtained from experimental tests performed using only the piston-rod assembly with removed valves. The applied friction model uses the hyperbolic tangent function tanh(.) which provides a smooth switch of friction force similar to experimental data. The friction between floating piston and pressure tube is omitted because it is significantly smaller than F_{fric}.

3.2. Model of flow restrictions

Compression and rebound valves are adjusted to ensure customer specifications regarding force and durability expectations. The force level is given as a function of force versus velocity while durability is typically specified in the number of sinusoidal cycles of given amplitude which damper has to withstand. The flow through a valve system can be modelled either in an (i) analytical or (ii) experimental manner. This paper takes into account the experimental model (static pressure-flow curves) because of its focus on a general vibration model and lack of space for details about a physical model valve system. On the other hand, a physical model creates challenges in modelling where a set of values of phenomenological parameters is required, such as discharge coefficients dependent on variable geometry when the valve opens/closes. Interested readers can find details about static and dynamic models of valve systems in the references [12]. The measured static pressure-flow characteristics of all restrictions are required in the model to capture the relationship between a pressure drop Δp across the considering valve assembly and volumetric flow rate q through the valve assembly.

3.3. Flow model

Changes in the oil mass in the compression and rebound chambers are obtained using the following mass flow equations

$$\dot{m}_{reb} = (q_{piston,com} - q_{piston,reb}) \cdot \rho_{reb,emu\_com} \tag{4}$$

where: m, $\dot{m}$, q, and ρ represent mass, mass flow rate, volumetric flow rate, and density respectively. The density of the oil flowing through the valves is obtained as an average value of the oil density in the connected chambers. The maximal calculated change in the density of the oil is less than 0.35% if the differential pressure over the valve does not exceed 10 bars [14]. Rearranging mass flow equation and knowing that

$$\rho_{reb,emu\_com} = \tfrac{1}{2} \cdot (\rho_{reb} + \rho_{emu\_com}) \tag{5}$$

$$\rho_{reb} = \frac{m_{reb}}{V_{reb}} \tag{6}$$

the following differential equations are obtained

$$\dot{m}_{reb} = \frac{1}{2} \cdot q_{piston} \cdot \frac{m_{reb}}{V_{reb}} + \frac{1}{2} \cdot q_{piston} \cdot \rho_{emu\_com} \tag{7}$$

where:

$$q_{piston} = q_{piston,com} - q_{piston,reb} \tag{8}$$

The volume V_{reb} depends on geometry and their initial values and are determined as follow

$$V_{reb} = V_{reb\_ini} + A_{reb} \cdot (x_{rod} - x_{tube}) \tag{9}$$

The average density of the oil-gas emulsion in the compression chamber ρ_{emu_res} is a function of the average density of the oil-gas emulsion in compression oil chamber and gas in the compression gas chamber ρ_{com}.

$$\rho_{emu\_com} = f(\rho_{com}) \tag{10}$$

Average density ρ_{com} can be calculated with formula

$$\rho_{com} = \frac{m_{gas\_com} + m_{emu\_com}}{V_{com}} \tag{11}$$

$$m_{emu\_com} = m_{emu} - m_{reb} \tag{12}$$

$$V_{com} = V_{com\_ini} + A_{com} \cdot (x_{tube} - x_{rod}) \tag{13}$$

where:
m_{gas_com} - the mass of free gas in the compression gas chamber [kg];
m_{emu_com} - the mass of the oil-gas emulation in the compression oil chamber [kg];
m_{emu} - the mass of the oil-gas emulation in whole mono-tube damper [kg];
m_{reb} - the mass of the oil-gas emulation in the rebound chamber [kg];
V_{com} - the volume of the compression chamber (sum of oil and gas chamber) [m^3].

Volumetric flows q thought piston depends on pressure drop Δp and are given as static characteristics.

$$\begin{aligned} q_{piston,com} &= f(\Delta p_{com,reb}); \\ q_{piston,reb} &= f(\Delta p_{reb,com}); \end{aligned} \tag{14}$$

$$\Delta p_{a,b} = p_a - p_b; \quad a,b = \{com, reb\} \tag{15}$$

Pressures p_{com}, p_{reb} can be determined as a function of density, as follows:

$$p_a = f(\rho_a); \quad a = \{com, reb\} \tag{16}$$

3.4. Two-phase flow model taking the aeration effect into account

One of the most important negative contributors to the low-frequency (lag of damping force) and high-frequency (excessive vibrations) performance of shock absorbers is the aeration effect. The delay in the build-up of damping force (pressure in the chambers) and the hysteresis loop in the force-velocity response is attributable to (i) fluid compressibility, or (ii) the existence of either a gas (aeration) or liquid vapour phase (cavitation) at certain stages of the stroking cycle. The presence of entrapped air or liquid vapour results in a large piston displacement before a significant pressure drop across the piston. Aeration and cavitation are complex phenomena which depend on a few factors, e.g. the purity of the liquid and the rate at which the liquid is stressed. The model proposed in the paper only takes into account the aeration effect using a homogeneous oil-gas model. The homogeneous model assumes that the gas and the liquid have the same velocity and are in the same thermal equilibrium. Additionally, the solubility of gas in liquid is constant and equal in all chambers. The solubility of the gas in the oil can be measured with the χ value. The χ value is a ratio of the mass of gas to the total mass of oil and gas. Using an empirically calculated (based on Henry's law), or obtained by experiment, value of χ, the density ρ of a homogeneous gas-oil mixture can be calculated with use of the following formulas:

$$\rho_{reb} = \rho_{emu\_reb} = \left(\frac{\chi}{\rho_{gas\_reb}} + \frac{1-\chi}{\rho_{oil\_reb}} \right)^{-1} \tag{17}$$

Using the above expressions, the pressure-density dependency in the rebound chamber can be determined in a few steps. In the first step, the values of pressure are calculated for assumed oil densities

$$m_1 \cdot \ddot{x}_{rod} + c_{TM} \cdot (\dot{x}_{rod} - \dot{x}_{TM}) + k_{TM} \cdot (x_{rod} - x_{TM}) = F_d \tag{18}$$

Next, the density of gas dissolved in the oil is determined

$$\rho_{gas\_reb} = \frac{p_{reb} \cdot M}{R \cdot T} \tag{19}$$

where: M, R and T are the gas constant, the temperature, and the molar mass of gas respectively. Finally Equations (17), (18) and (19) are solved and the following dependency is obtained

$$p_{reb} = f(\rho_{reb}). \tag{20}$$

The obtained pressure-density function of a homogeneous gas-oil mixture is presented in Fig. 4. One can observe that a linear dependency for oil (fluid compressibility) and a linear dependency for gas (isothermal compressibility) create a non-linear dependency for their mixture.

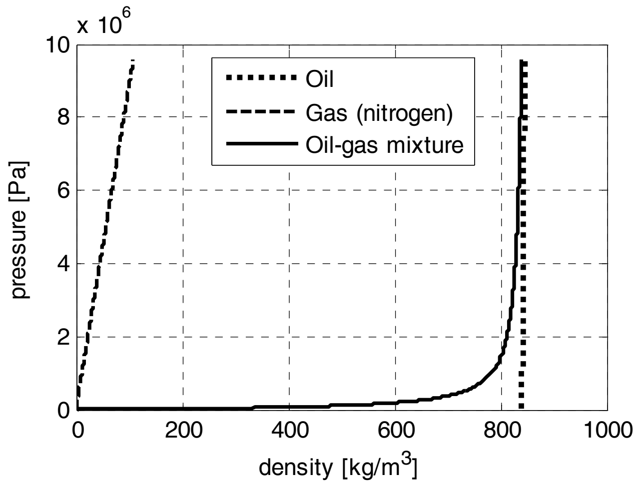

Fig. 4. Density-pressure relation for gas (isothermal), oil, and gas-oil mixture

Determination of pressure-density dependencies in the compression chamber requires additional steps. As the first step, the total mass of gas (gas in the compression gas chamber and gas in the gas-oil emulsion) is obtained:

$$m_{gas} = \frac{p_{ini} \cdot V_{gas\_ini} \cdot M}{R \cdot T} \tag{21}$$

After that, the mass of gas in compression gas chamber is estimated

$$m_{gas\_com} = m_{gas} - \chi \cdot m_{oil} \tag{22}$$

For the expected range of oil density ρ_{oil_com} independent variable assumed for calculation of final dependencies (23), the pressure p_{com} in the compression chamber is calculated as follows

$$p_{com} = p_{ini} + K \ln\left(\frac{\rho_{oil\_com}}{\rho_{ini}}\right) \tag{23}$$

Next, the density of gas in the compression gas chamber and gas in the gas-oil emulsion in the compression oil chamber is determined

$$\rho_{gas\_com} = \frac{p_{com} \cdot M}{R \cdot T} \tag{24}$$

The average density of the oil-gas emulsion in the compression chamber is calculated based on the obtained densities, as follows

$$\rho_{emu\_com} = \left(\frac{\chi}{\rho_{gas\_com}} + \frac{1-\chi}{\rho_{oil\_com}}\right)^{-1} \tag{25}$$

Using the calculated values, the volume V_{emu_com} and the mass m_{emu_com} of the homogeneous oil-gas emulsion in the compression oil chamber are:

$$V_{emu\_com} = V_{com} - \frac{m_{gas\_com}}{\rho_{gas\_com}} \tag{26}$$

$$m_{emu\_com} = V_{emu\_com} \cdot \rho_{emu\_com} \tag{27}$$

Finally, the average density of the homogeneous oil-gas emulsion in the compression oil chamber and gas in the compression gas chamber is calculated using the following formula:

$$\rho_{com} = \left(\frac{\chi_{com}}{\rho_{gas\_com}} + \frac{1-\chi_{com}}{\rho_{emu\_com}}\right)^{-1} \tag{28}$$

where

$$\chi_{com} = \frac{m_{gas\_com}}{m_{emu\_com} + m_{gas\_com}} \tag{29}$$

After these steps, the following dependencies are available:

$$\begin{aligned} p_{com} &= f(\rho_{com}); \\ \rho_{emu\_com} &= f(\rho_{com}). \end{aligned} \tag{30}$$

4. Adjustment of model parameters

A model of a technical system to be adjusted to operational data is represented as a set of nonlinear state-space equations formulated in the continuous-time domain as follows

$$\begin{aligned} \frac{d}{dt}x(t) &= f\big(t, x(t), u(t), w(t); \theta\big) \\ y(t) &= h\big(t, x(t), u(t), v(t); \theta\big) \\ x(0) &= x_0 \end{aligned} \tag{31}$$

where vector f(.) is a nonlinear, time-varying function of the state vector x(t) and the control vector u(t), while vector h(.) is a nonlinear measurement function; w(t) and v(t) are sequences of independent random variables and θ denotes a vector of unknown parameters. In nonlinear systems, the state vectors and the measurements vectors are not Gaussian distributed. The predictor resulting from the model (31) takes the form:

$$\hat{y}(t\,|\,\theta) = g(t, Z^{t-1}; \theta) \tag{32}$$

where Z^N ={y(t), u(t), k=1,…N} while the prediction error equation has the form

$$\varepsilon(t,\theta) = y(t) - g(t, Z^{t-1}; \theta) \tag{33}$$

The sum of squared errors is used as an error criterion. This problem is known in numerical analysis as "the nonlinear least-squares problem" [15]. The objective of the estimation is to minimize the $V_N(\theta)$ error function by means of an iterative numerical technique. The error function $V_N(\theta)$ has the form

$$V_N(\theta, Z^N) = \frac{1}{N}\sum_{t=1}^{N}\frac{1}{2}\varepsilon^2(t,\theta) \tag{34}$$

Three methods of minimizing the error function (34) are feasible: direct search, first-order, and second-order methods [15]. Direct search methods use only the value of the function to find the minimum. The first-order method uses the information provided by the first derivative (gradient) of the error function, while the second-order method uses both, information regarding the first and the second order derivatives (gradient and Hessian form) of the error function.

The 'idnlgrey' model structure available in System Identification Toolbox (Matlab [16]) was used to implement the simulation model introduced in Section 3. The model structure supports the models in a form of nonlinear ordinary differential or difference equation. The equations have to be formed as a set of first-order differential or difference equations. The model has two inputs which are the signals of displacement and velocity and one output which is the rod force. It is therefore classified as MISO structure. The adjustment algorithm simulates the model several times trying various parameter values to reduce the prediction error. The following algorithm properties can affect the quality of the results:

- solver settings;
- optimization method;
- gradient options;
- other specified algorithm properties.

A one damper cycle including compression and rebound stroke was used to adjust the model with Method A and B (Fig. 5).

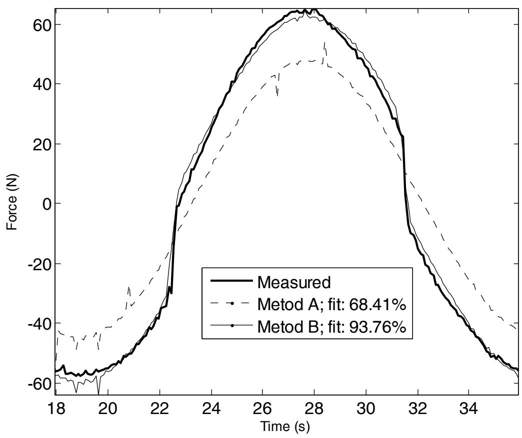

Fig. 5. Adjustment results for two parameters present in the friction model (test velocity v=0.018 m/s)

Method A is a trial-end-error approach, while Method B is the model adjustment using a second order optimization technique, i.e. Levenberg-Marquardt. Two model parameters were adjusted, namely: maximal force and reference velocity included in the friction model (Table 1).

Table 1.
Model adjustment results

	(min; max)	Method A (trial-and-error)	Method B (optimization)
F_{fric_max}	(0.1;100)	0.1	14.84
v_{ref}	(1E-6; 1)	1E-5	1.53E-5

System Identification Toolbox provides several variable-step and fixed-step solvers for simulating 'idnlgrey' models, e.g. 'ode45'. However, 'ode23tb' method was chosen to accelerate the simulation and shorten calculation time.

5. Summary

The paper proposes and demonstrates an automated method towards adjustment of a first-principle model of a hydraulic damper. The model is represented by nonlinear state-space equations having geometrical and physical parameters deduced from available documentation. The model free parameters, so-called adjustable phenomenological parameters (friction model parameters) are adjusted based on the initial values from measurement data.

The proposed adjustment method with the use of error function minimization algorithm shows required performance to achieve accuracy higher than 90% compared to trial-and-error manual model adjustment process to measurement data.

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