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The identification of degenerated systems in the impact energy dissipation process

M. Bocian [a], K. Jamroziak [b,]*

[a] Institute of Materials Science and Applied Mechanics,
Wroclaw University of Technology, ul. Smoluchowskiego 25, 50-370 Wrocław, Poland

[b] The Tadeusz Kosciuszko Military Academy of Land Forces,
ul. Czajkowskiego 109, 51-150 Wrocław, Poland

* Corresponding e-mail address: krzysztof.jamroziak@wso.wroc.pl

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Analysis and modelling

ABSTRACT

Purpose: The article presents an analysis of impact energy dissipation process with selected non-classical dynamic models.

Design/methodology/approach: Identification of impact energy dissipation phenomena in mechanical systems with a layered structure (eg, composite ballistic shields) is quite a challenge, because on the one hand it is sought to the model, whose parameters are as much as possible responsible for the energy dissipation, on the other hand, the number of parameters should be optimized. Searched model should be reduced to a simple description of the whole phenomenon and completely imitate entire mechanical system. Description of the impact energy dissipation was modeled with selected degenerated systems in this case. Models were subjected to hammer extortion the specified impulse of force. The mathematical description of pulsed extortion was carried out by using the energy and balance equation of power. Verification of mathematical identification equations for selected model parameters was performed by computer simulation technique.

Findings: This is original analytical method, which uses the degenerated systems in various configurations. It involves the use of specially derived identification equations, which are described by the decrease of potential energy of the system during the vibrations induced by a single impulse load.

Research limitations/implications: Method of identification requires the use of appropriate input function. Input function could be a periodic type or a type of step function.

Practical implications: Estimation of the energy consumption objects in terms of method of identifying the parameters of the model.

Originality/value: Presented work includes the identification of piercing the ballistic shield, and it is a part of work on the implementation of the degenerated models to describe these phenomena.

Keywords: Computational mechanics; Impact load; Degenerated model; Equations energy and potency balance

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1. Introduction

One of the essential elements in engineering applications is the identification of the impact energy dissipation by developing a model with carefully selected procedures. Basing on the methodological literature for identification of mechanical systems in the development of models, their extortion, including impulse ones and testing of a model's reaction susceptibility to extortion is described by many studies [1-5]. Most of research concerned with developing mathematical algorithm of implementation of both task analysis and synthesis of simple and complex mechanical and mechatronic systems [6-8]. The need for modelling dynamic phenomena of machine elements and structures results with the determination of the fatigue endurance, which is a consequence of the application of new generation materials [9-16].

Modelling of the penetrable ballistic shield is such an example [17-20]. Currently, there are many well-known formulas for describing of impact energy dissipation, which have been extensively presented in following studies [21,22]. However the penetration and dissipation process of the composite shields remains still a challenge during researches for the optimal identification procedures. Development of identifying procedures to non-classical dynamical models which include degenerate models [23,24] is one of the methods.

Those models can be characterized by having, inter alia, non-linear segments, which have no closed mathematical analysis procedures for now.

Therefore, it seems that the atypical approach to the matter in the form of the advanced energy and power balance Equations is a novel approach in the field of non-linear dynamical systems theory, as proved in following studies [25-27]. It should be noted that these Equations are compiled (met) at a certain vibration conditions. This is especially important in the design of optimal ballistic shields, as well as any other system subjected to the piercing processes. The analysis of these Equations is described in the following section.

2. Description and analysis of the models impact energy dissipation

Energy dispersion analysis of mechanical systems was subjected by two dynamic models, which the diagrams are shown in Fig. 1. Modelling of the impact energy dissipation process during an impact of projectile into light ballistic shield was presented as follows:

- for 1a system the existence of Maxwell's element has been assumed, which cooperates in parallel with elastic element (c) and dry friction element (h),
- for 1b system the existence of Maxwell's element has been assumed, which cooperates in parallel with the elastic element (c) and the damping element (k).

It has been assumed that the analysis of degenerated systems is based on the derivation of energy and power balance Equations. These Equations are characterized by the following features:

- system of two differential Equations are brought to a single Equation of the third rank,
- the Equations are multiplied by:
  - elementary displacement dx for the energy balance Equation;
  - elementary velocity dv for the of power balance Equation;
- integral Equations are within the limits of time equal to the period of vibration or its multiplicity.

It is adopted by two variables describing the motion of the analyzed systems, namely:

- variable ξ describing the motion of a fictitious mass $m_f = 0$;
- variable x describing the displacement of a real mass m of the system.

The extensive description of the implementation and application of the energy and power balance Equations in mechanical systems have been presented in study [28], and certain assumptions referred in studies [29,30]. Thus, the idea of identification with the use these Equations can be illustrated by block wise (Fig. 2).

2.1. Impact energy dissipation analysis by degenerated model with dry friction

In the first system, which is with the dry friction element (h) and elastic element (c) in accordance with conditions of dynamic equilibrium differential Equations of motion for the variables ξ, x take the form of:

$$m\ddot{x} + hSgn\dot{x} + cx + c_0(x-\xi) = p \tag{1}$$

$$k_0\dot{\xi} = c_0(x-\xi) \tag{2}$$

By eliminating of variable ξ Equations can be written as a single Equation:

$$m\ddot{x} + hSgn\dot{x} + cx + \frac{k_0}{c_0}\left[m\dddot{x} + hSgn\dot{x}\ddot{x} + (c_0+c)\dot{x} - \dot{p}\right] = p \tag{3}$$

It is assumed that the energy dissipation in the time interval from

$t = 0$ (the moment of impact) to time t_k (time for stopping the projectile) for:

$$t \in (0, t_k) \rightarrow \dot{x} > 0 \text{, and } Sgn\dot{x} = 1 \tag{4}$$

Finally, Equation (3) takes the form:

$$m\ddot{x} + h + cx + \frac{k_0}{c_0}\left[m\dddot{x} + h\ddot{x} + (c_0+c)\dot{x} - \dot{p}\right] = p \tag{5}$$

It is assumed that in case of impact ballistic (projectile into ballistic shield) struck element has some dissipative-elastic properties. For further analysis, it was assumed that on weight of the striking force acts only influence of the material S. This force depends on the velocity and displacement (position of the projectile in the material). According to the second law of dynamics, this condition can be written as:

$$m\ddot{x} + S = p(t) \tag{6}$$

where:

x - displacement,

$p(t)$ - extortion force,

S - symbolically power of influence the material on the bullet expressed as a linear $S = k\dot{x} + cx$,

k - viscous damping coefficient,

c - coefficient of elasticity.

Based on analysis of the 1a model, the impact force of material S on striking mass m can be described as a function:

$$S = (x, \dot{x}, \ddot{x}, \dot{p}) = h + cx + \frac{k_0}{c_0}\left[m\dddot{x} + h\ddot{x} + (c_0+c)\dot{x} - \dot{p}\right] = p \tag{7}$$

Expression (7) describing the strength S is significantly different from the one usually used in practice. Dissipation energy resulting from the expression (7) for cyclic vibrations one can be estimated from the energy balance Equation. According to study [28], the energy balance Equation can be expressed as:

$$h(x_k - x_0) + \frac{k_0 m}{c_o}\alpha_a^v + \frac{k_0}{c_0}(c_0 + c)\alpha_x^v + h\frac{k_0}{c_0}\alpha_x^a - \frac{k_0}{c_0}\alpha_p^v = \alpha_x^p \tag{8}$$

It is noted that for the extortion pulse terminating in the range $t_0 < t_k$ following units of Equation (8) take the value zero, i.e.:

$$\frac{k_0}{c_0}h\ddot{x} = \frac{k_0}{c_0}h\alpha_x^a = 0 \tag{9}$$

$$-\frac{k_0}{c_0}\dot{p} = -\frac{k_0}{c_0}\alpha_p^v = 0 \tag{10}$$

and eventually the energy balance Equation takes the form:

$$h(x_k - x_0) + \frac{k_0 m}{c_o}\alpha_a^v + \frac{k_0}{c_0}(c_0 + c)\alpha_x^v = \alpha_x^p \tag{11}$$

In analogy power balance Equation takes the form:

$$h(v_k - v_0) + \left(m + \frac{k_0}{c_0}\right)\alpha_a^v - \frac{k_0}{c_0}\alpha_p^a = \alpha_v^p \tag{12}$$

Equations (11) and (12) after arranging can be presented:

$$A_1 h + B_1\alpha_a^v + B_2\alpha_x^v = \alpha_x^p \tag{13}$$

where:

$$A_1 = x_k - x_0, \quad B_1 = \frac{mk_0}{c_0}, \quad B_2 = \frac{k_0}{c_0}(c_0 + c)$$

$$A_2 h + B_1\alpha_a^v + B_3\alpha_x^v = \alpha_x^p \tag{14}$$

where:

$$A_2 = v_k - v_0, \quad B_1 = \frac{mk_0}{c_0}, \quad B_3 = -\frac{k_0}{c_0}$$

All of the variables α in the Equations are equal to the fields contained within the loop formed by the respective depending periodic signals $v(x)$, $a(v)$, $a(p)$, $p(v)$, $p(x)$. The basic condition of these Equations is periodicity of the measured signals.

2.2. Impact energy dispersive analysis by degenerated model with viscous damping

In the arrangement of elements (elastic c and damping k), shown in Fig. 1b, in accordance with the dynamic equilibrium conditions the differential Equations of motion for the variables ξ, x take the form of:

$$m\ddot{x} + k\dot{x} + cx + c_0(x - \xi) = p \tag{15}$$

$$k_0\dot{\xi} = c_0(x - \xi) \tag{16}$$

Proceeding analogically to the analysis of model 1a Equation (3) brought to a single Equation of form:

$$m\ddot{x} + k\dot{x} + cx + \frac{k_0}{c_0}\left[m\dddot{x} + k\ddot{x} + (c_0 + c)\dot{x} - \dot{p}\right] = p \tag{17}$$

The impact of the material S striking mass m can be described as a function of:

$$S = S(x, \dot{x}, \ddot{x}, \dot{p}) = k\dot{x} + cx + \frac{k_0}{c_0}\left[m\dddot{x} + k\ddot{x} + (c_0 + c)\dot{x} - \dot{p}\right] = p \tag{18}$$

The Equation (17) is developed analogically to the previous instance. In this case the methodology of proceeding gives the energy balance Equation as follows:

$$\left(k + k_0 + k_0\frac{c}{c_0}\right)\alpha_x^v + \frac{k_0 m}{c_0}\alpha_a^v - \frac{k_0}{c_0}\alpha_p^v = \alpha_x^p \tag{19}$$

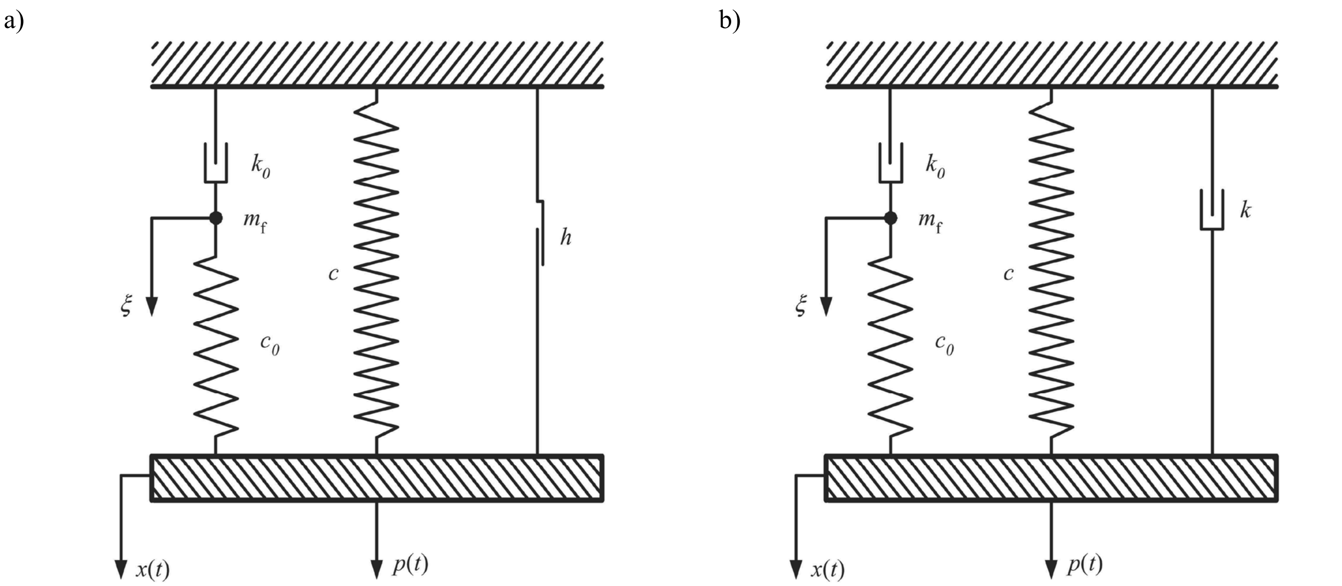

Fig. 1. The dynamic model of energy dissipation: a) degenerated model with the dry friction, b) degenerate model with the damping element

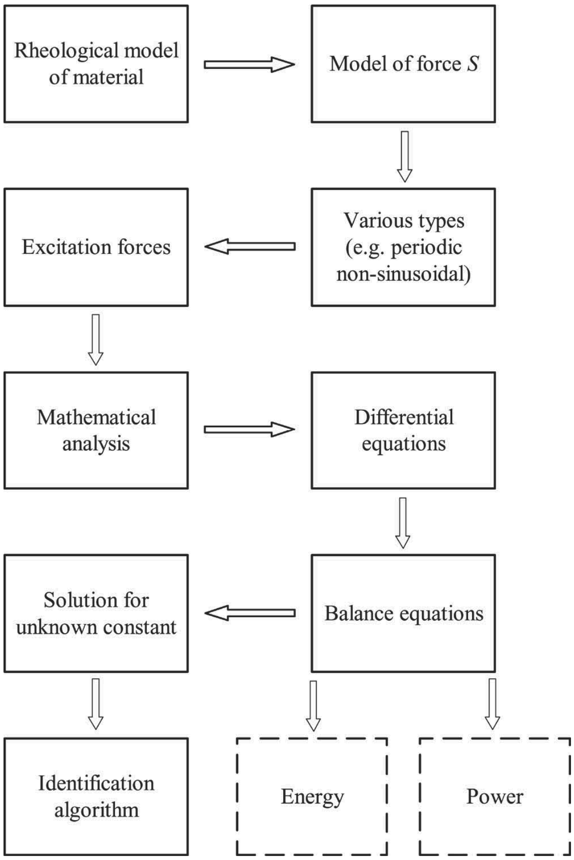

Fig. 2. Idea of identification with the use of Equations of energy balance and power

Power balance Equation takes the form:

$$\left(m+\frac{k_0 k}{c_0}\right)\alpha_v^a + c\alpha_v^x - \frac{k_0}{c_0}\alpha_p^a = \alpha_v^p \tag{20}$$

Equations (19) and (20) after arranging can be presented:

$$X_1\alpha_x^v + X_2\alpha_a^v + X_3\alpha_p^v = \alpha_x^p \tag{21}$$

where:

$$X_1 = k + k_0 + k_0\frac{c}{c_0}, \qquad X_2 = \frac{mk_0}{c_0}, \qquad X_3 = -\frac{k_0}{c_0}$$

$$X_4\alpha_v^a + X_5\alpha_v^x + X_3\alpha_p^a = \alpha_v^p \tag{22}$$

where:

$$X_4 = m + \frac{kk_0}{c_0}, \qquad X_5 = c, \qquad X_3 = -\frac{k_0}{c_0}$$

The derived energy balance Equation (11), (19) and the power balance Equation (12), (20) are characterized by the size of the impact energy dissipation through the constant element of degenerated and dry friction parameter values (model 1a), viscous damping (model 1b). Right part of the Equations describes, respectively:

- the work done by an external force $p(t)$ on displacement x within a single complete period (variable α);
- work per unit time (power) done by this force (variable α).

The conducted analysis of dynamic models with a usage of energy and balance and power balance Equations reflects the theoretical background of the impact energy dissipation. In order to describe the real system with those Equations model should be subjected with loads in the form of impulses at regular intervals and recorded the responses assuming the following condition:

$$x(t) = x(t+T) \tag{23}$$

Answers should reflect the actual state, if so it is possible considered sought models as valid. To prove the assumptions in this field of theoretical analysis, models were subjected to computer simulation, which was the subject of analysis in the next step of work.

3. Simulation studies and analysis of the results

The complexity of the analyzed identification Equations models, the requires the use age of numerical techniques. Numerical techniques such as Simulink software allows for quick verification of identification Equations and assumed the model parameters, and in further stage the optimization, also in terms of the actual object (ballistic shields).

The simulation tests were carried out by substituting to Equations (11), (12), (19), (20) of the analyzed models by the following values: k=480 [kg/s], c=30000 [kg/s^2], c_0 = 20000 [kg/s^2], m=40 [kg], k_0=406 [kg/s], h=5 N.

In all cases the simulated impulse extortion was (Fig. 3):

- for $t < 0{,}001$;
- where the extorting force $p(t)$ was modeled according to the function $p(t) = ASin(1000\pi t)$.

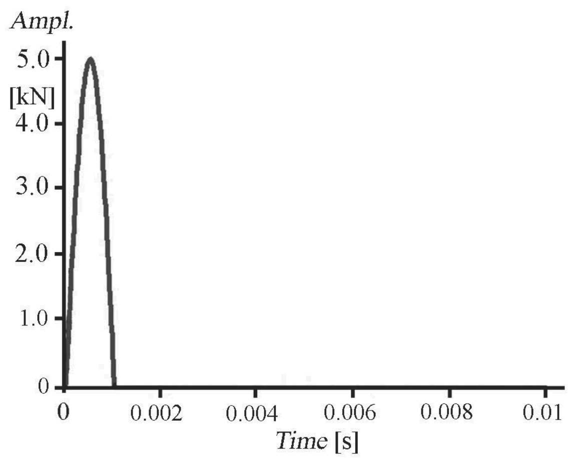

Fig. 3. The shape of the exciting force $p(t)$

The simulation of the degenerated model with viscous damping (model 1b) was carried out at several extortion assuming the amplitude $Ampl$=5 kN and 500 Hz frequency, taking into account the above parameters of the model at h=0. Then amplitude was changed to $Ampl$=15 kN, next $Ampl$=25. The integration was carried out for a time from t_0 to t_k if v<0 assuming the $v(t_0) = 0$ and $v(t_k) = 0$. It is the range, where the extortion force is set to zero ($p(t) = 0$). The analyzes are illustrated graphically in Fig. 4.

The simulation with dry friction (model 1a) was performed for similar conditions by adopting the parameter h=5. Analysis is presented as graphical form in Fig. 5.

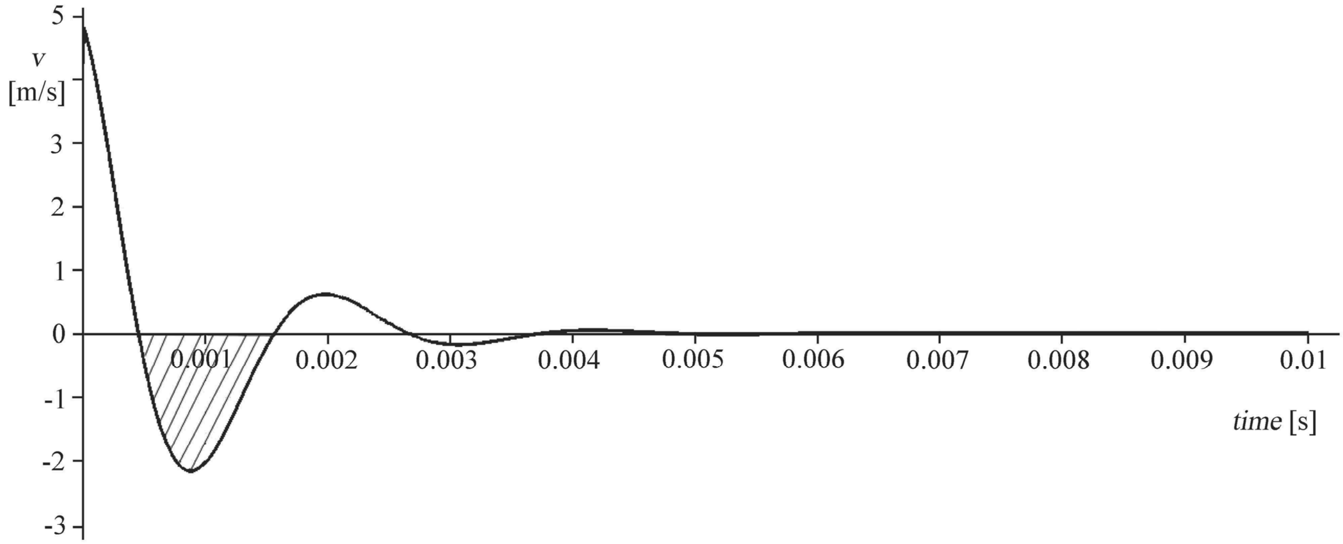

Fig. 4. Velocity as a function of time to the extortion impulse of the model (Fig. 1b)

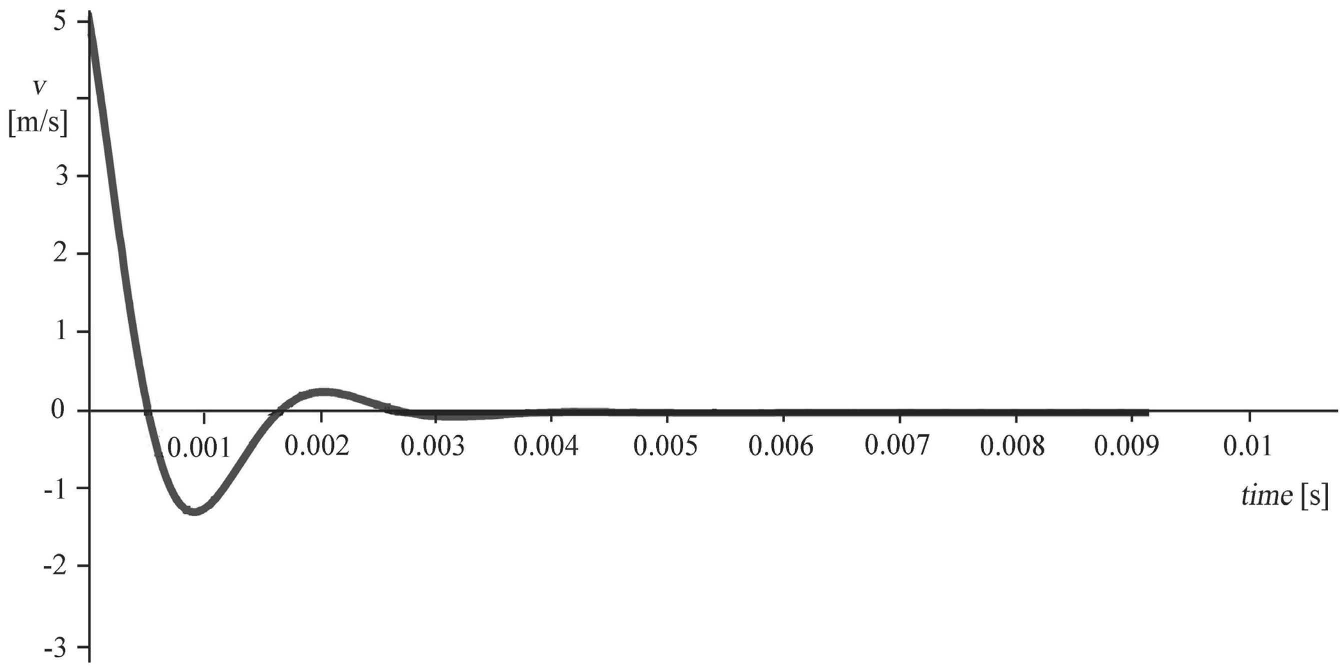

Fig. 5. Velocity as a function of time to the extortion impulse of the model (Fig. 1a)

In applying linear regression analysis fixed coefficients were calculated α for power and balance Equations. The data is summarized in Table 1 - model 1b, Table 2 - model 1a.

Table 1.
The parameters assumed and derived from the linear regression for model 1b

Parameter	Assumed	Derived
	Energy balance	
X_1	528.9	550.0
X_2	0.33	0.36
X_3	0.0083	0.009
	Power balance	
X_4	40.10	35.77
X_5	30000	30011

Table 2.
The parameters assumed and derived from the linear regression for model 1a

Parameter	Assumed	Derived
	Energy balance	
A_1	5.0	5.02
B_1	2.7067e^{-5}	2.7166e^{-5}
B_2	0.0535	0.0499
	Power balance	
A_2	5.0	4.98
B_3	0.0017	0.00171

In this way it has been adjusted to determine the coefficients of the identification Equations, complex expressions of stiffness, suppression, and mass which are occurring in the system. The final element of identification (Fig. 2) is to lead to determination of the specific parameters of the model. Estimation of model parameters can be carried out by the methods, chich already presented in the study [25] by the quasi-static loads or continue using the energy and power balance Equations. To do this, a simple Kelvin model supposed to be taken which will allow us to estimate a parameter c. For example this may be a model as shown in Fig. 6. Energy balance Equation, which is achieved by starting from the motion Equations mass m of dynamic extortion:

$$m\ddot{x} + k\dot{x} + cx = p(t) \tag{24}$$

Energy balance Equation is obtained by multiplying both sides of Equation (24) by dx, then integrating in the period T, and this results with:

$$m\int_T \ddot{x}dx + k\int_T \dot{x}dx + c\int_T xdx = \int_T p(t)dx \tag{25}$$

Integrating by parts Equation (25) we obtain the following order:

$$m\int_T \ddot{x}dx = \int_T \frac{d\dot{x}}{dt}\dot{x}dt = 0 \tag{26}$$

$$m\int_T \dot{x}\cdot\dot{x}dt = \alpha_x^v \tag{27}$$

$$\int_T x\cdot\dot{x}dt = \int_T xdt = 0 \tag{28}$$

$$\int_T p(t)\cdot\dot{x}dt = \alpha_x^p \tag{29}$$

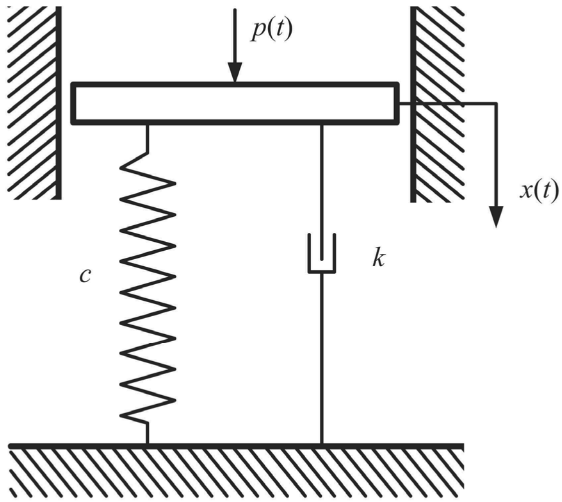

Fig. 6. Kelvin model related to the determination of the parameter c

The energy balance Equation takes the form of:

$$k\cdot\alpha_x^v = \alpha_x^p \tag{30}$$

Power balance Equation is obtained by multiplying both sides of Equation (24) by dv ($dv = \ddot{x}dt$) and integrating in the period T, and this results with:

$$m\int_T \ddot{x}\cdot\ddot{x}dt + k\int_T \dot{x}\cdot\ddot{x}dt + c\int_T x\cdot\ddot{x}dt = \int_T p(t)\cdot\ddot{x}dt \tag{31}$$

By the analogy with the previous Equation (31) integration over the parts successively obtain with:

$$\int_T \ddot{x}\cdot\dot{x}dt = \alpha_v^a \tag{32}$$

$$\int_T x\cdot\ddot{x}dt = \alpha_v^x \tag{33}$$

$$\int_T p(t)\cdot\ddot{x}dt = \alpha_v^p \tag{34}$$

Power balance Equation takes the form of:

$$m\cdot\alpha_v^a + c\cdot\alpha_v^x = \alpha_v^p \tag{35}$$

Some constants (α) were calculated by using regression methods from Equations (30) and (35) with a known parameter of mass m. Parameter c was calculated from Equation (35), and the parameter k was calculated from Equation (30). The remaining constants were estimated after substituting the obtained parameters c, k to (13), (14), (21), (22). Obtained results present a problem in terms of quality.

4. Conclusions

Presented part of research concerns the analysis of energy consumption in dynamic rheological models during ballistic impact process. The analysis adopted two dynamic models, which are a common part of the Maxwell element. The first one is a model describing vibrations of mechanical systems with dry friction, and the second is a model describing vibrations of mechanical systems viscous damping.

The models were also subjected to a computer simulation to verify the assumptions made in the analysis about validity of the mathematical description in the phenomenon of impact energy dissipation.

The result of the simulations was to obtain a response time for the impulse force $p(t)$. It was assumed that the time points t_0, t_k displacement $x(t)$ should take extreme values for the time points for which the velocity $v(t)$ must take zero values. Simulation for the assumed moments of time (Figs. 4-5) velocity $v(t)$ reaches zero values, as shown.

The results also prove some differences. In the analysed period of time the model with dry friction element has more similar form to real results (Fig. 7). Its composition of the description of the impact energy dissipation has shown increased energy consumption compared to the model with viscous friction element.

An analogy can be seen from the results of recorded during the studies of piercing material on the firing position that for extreme values $x(t_0)$ and $x(t_k)$ of $v(t)$ take the zero value (Figs. 7-8). Model presents the phenomenon of impact energy dissipation by the energy and power balance Equations and brings closer a physical phenomenon occurring in the process of piercing the material.

It can be assumed that this methodology can be used to describe the mathematical and graphs obtained from the simulation describes qualitatively this issue. In terms of volume it should strive to illustrate by a number of studies that the analysis will be continued and presented in subsequent papers.

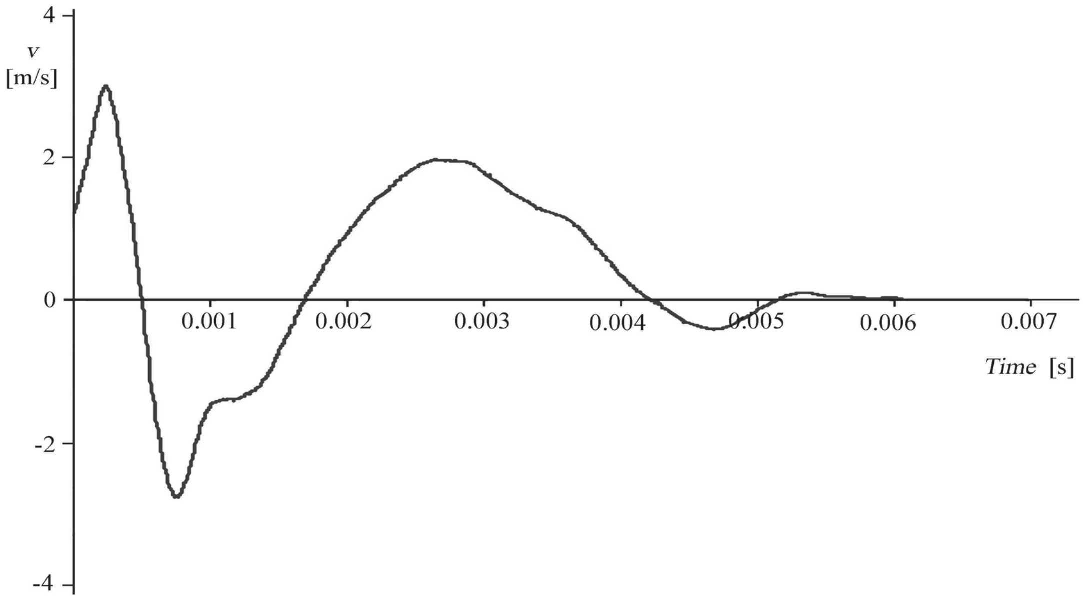

Fig. 7. The chart of speed (shield-bullet) as a function of time during the penetration of the cover with 9 mm Parabellum bullet at the impact speed $v_d = 352$ m/s

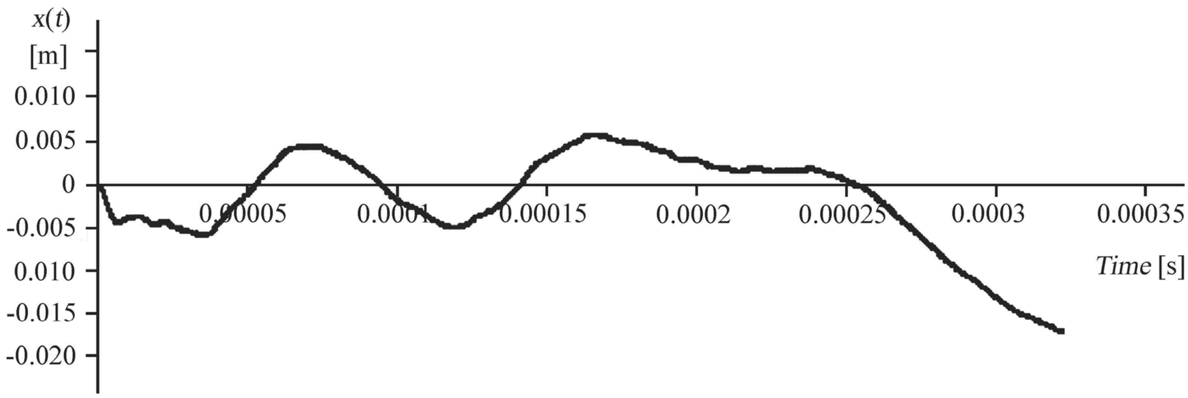

Fig. 8. The chart of shield displacement in the point of impact during penetration of the cover with 9 mm Parabellum bullet at the impact speed $v_d = 352$ m/s

References

[1] K. Arczewski, J. Pietrucha, J.T. Szuster, Vibration of physical systems, Publishing Office of Warsaw University of Technology, Warsaw, 2008 (in Polish).

[2] A. Buchacz, Modeling, synthesis, modification, sensitivity and analysis of mechanic and mechatronic systems, Journal of Achievements in Materials and Manufacturing Engineering 24/1 (2007) 198-207.

[3] A Buchacz, Calculation of flexibility of vibrating beam as the subsystem of mechatronic system by means the exact and approximate methods, Proceedings in Applied Mathematics and Mechanics 9/1 (2009) 373 - 374.

[4] K. Jamroziak, The dry friction influence on dissipation of impact energy, Selected Engineering Problems 2 (2011) 139-144.

[5] A. Wróbel, Kelvin Voigt's model of single piezoelectric plate, Journal of Vibroengineering 14/2 (2012) 534-537.

[6] K. Białas, Application of mechanical and electrical elements in reduction of vibration, Journal of Achievements in Materials and Manufacturing Engineering 52/1 (2012) 31-38.

[7] A. Buchacz, Formulating of reverse task of chosen class of mechatronic systems, Journal of Achievements in Materials and Manufacturing Engineering 54/1 (2012) 75-82.

[8] S. Zolkiewski, Dynamic flexibility of the supported-clamped beam in transportation, Journal of Vibroengineering 13/4 (2011) 810-816.

[9] A. Baier, M. Majzner, Analysis of composite structural elements, Journal of Achievements in Materials and Manufacturing Engineering 43/2 (2010) 557-585.

[10] A. Baier, M. Majzner, Modelling and testing of composite fiber, Design and Construction Engineering 9/39 (2010) 22-28.

[11] M. Rojek, Methodology of diagnostic testing of polymeric matrix laminate composite materials, Open Access Library, 2, 2011, 1-148 (in Polish).

[12] L.A. Dobrzański, A. Pusz, A.J. Nowak, Aramid-silicon laminatem materials with special properties - new perspective of its usage, Journal of Achievements in Materials and Manufacturing Engineering 28/1 (2008) 7-14.

[13] P. Fedeliński, Computer modelling and analysis of microstructures with fibres and cracks, Journal of Achievements in Materials and Manufacturing Engineering 54/2 (2012) 242-249.

[14] M. Grujicic, P.S. Glomski, T. He, G. Arakere, W.C. Bell, B.A. Cheeseman, Material modeling and ballistic-resistance analysis of armor-grade composites reinforced with high-performance fibers, Journal of Materials Engineering and Performance 18/9 (2009) 1169-1182.

[15] C.C. Liang, C.Y. Jen, M.F. Yang, P.W. Wu, Resistant performance of performation in protective structures using a semi-empircal method with marine applications, Ocean Engineering 30 (2003) 1137-1162.

[16] G. Wróbel, S. Pawlak, G. Muzia, Thermal diffusivity measurements of selected fiber reinforced polymer composites using heat pulse metod, Archives of Materials Science and Engineering 48/1 (2011) 25-32.

[17] K. Jamroziak, Analysis of a degenerated standard model in the piercing process, Journal of Achievements in Materials and Manufacturing Engineering 22/1 (2007) 49-52.

[18] K. Jamroziak, Process description of piercing when using a degenerated model, Journal of Achievements in Materials and Manufacturing Engineering 26/1 (2008) 57-64.

[19] K. Jamroziak, M. Bocian, Identification of composite materials at high speed deformation with the use of degenerated model, Journal of Achievements in Materials and Manufacturing Engineering 28/2 (2008) 171-174.

[20] M. Kulisiewicz, M. Bocian, K. Jamroziak, Criteria of material selection for ballistic shields in the context of chosen degenerated models, Journal of Achievements in Materials and Manufacturing Engineering 31/2 (2008) 505-509.

[21] P. Bourke, Ballistic impact on composite armour, Cranfield University, 2007.

[22] D.E. Carlucci, S.S. Jacobson, Ballistic - theory and design of guns and ammunition, CRC Press, 2008.

[23] M. Kulisiewicz, S. Piesiak, The methodology of modeling and identification of mechanical dynamical systems, University of Technology, Wrocław, 1994 (in Polish).

[24] M. Kulisiewicz, Modelling and identification of nonlinear mechanical systems under dynamic complex loads, University of Technology, Wrocław, 2005 (in Polish).

[25] K. Jamroziak, Identification of the selected parameters of the model in the process of ballistic impact, Journal of Achievements in Materials and Manufacturing Engineering 49/2 (2011) 305-312.

[26] K. Jamroziak, B. Bocian, M. Kulisiewicz, Evaluation of a composite systems piercing with the use of degenerated models, High-Speed Tracked Vehicles 2 (2012) 41-50 (in Polish).

[27] K. Jamroziak, Description of loss of the impact energy on the example of the selected degenerate systems, International Journal of Proceedings Machine Buildings and Systems 3 (2012) 140-143.

[28] S. Piesiak, Identification of mechanical systems in domain nonlinear and degenerated dynamical models, University of Technology, Wrocław, 2003 (in Polish).

[29] M. Bocian, M. Kulisiewicz, S. Piesiak, Computer studies of a degenerate system in terms of applications of energy balance and power for complex harmonic excitations, SYSTEMS - Journal of Transdisciplinary Systems Science, 9 (2004) 120-127.

[30] M. Bocian, K. Jamroziak, M. Kulisiewicz, Determination of the chain-like non-linear multi-degree-of-freedom systems constant parameters under dynamical complex loads, Proceedings in Applied Mathematics and Mechanics 9/1 (2009) 397-398.