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Transmission and photonic band gaps in Fibonacci superlattices

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Properties

ABSTRACT

Purpose: The purpose of the article was to broaden the knowledge about the behavior of Fibonacci superlattices as filters electromagnetic waves.
Design/methodology/approach: Simulations of multi-layer systems is usually carried out by using two complementary methods. The first, matrix method which allows the study of the properties of structures using transmission maps and the second method used is the Finite-Difference Time Domain (FDTD) algorithm allows on the study of electromagnetic wave propagation in the structure.
Findings: It can be seen that the lighting of the filter with monochromatic light in the wavelength range of the band gap filter at the output causes propagation of low intensity in the range other than the wavelength of the incident beam.
Research limitations/implications: The simulation was not considered the impact of losses in the material.
Practical implications: Present clear differences depending on the polarization allow the use of superlattices as polarizers for specific ranges of wavelengths and angles of incidence.
Originality/value: Fibonacci superlattices have been pre-tested in. The purpose of the article was to broaden the knowledge about the behavior of these structures as filters electromagnetic waves with a wavelength range from the near infrared, the effect of the material surrounding the transmission and increasing knowledge of the formation of band gaps.
Keywords: Transmission; Multilayers; Superlattices; Aperiodic; LHM; RHM

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1. Introduction

Quasiperiodic multilayer systems because of their unique properties are used as filters of electromagnetic radiation and as photonic materials are intensively studied in many research centers all over the world [1-5]. There exists a phenomenon of the photonic band gap, and therefore electromagnetic waves of certain wavelengths do not propagate in the materials. The size and the occurrence of band gaps is strongly correlated with the type of materials used to build the structure, the structure of arrangement layers, their thickness and the type of environment in which the multilayer is to operate.

Well-developed technique for producing multilayer systems allows to obtain material having selected physical properties [6-32]. The use of superlattices as a filter of electromagnetic waves in optical multiplexers creates the need for filter design with specific areas of the propagation of electromagnetic waves and band gaps. In order to reduce the cost of research shall be carried out preliminary simulations of these structures. Possibly accurate and comprehensive analysis of various types of structures allows for a better understanding of the phenomena occurring in the material, and thus the design of multilayers with a much better performance characteristics.

Simulations of multi-layer systems is usually carried out by using two complementary methods. The first, matrix method [2, 42], allows the study of the properties of structures using transmission maps. They allow you to know the structure of the propagation of electromagnetic waves by multilayer system for a given wavelength range and any angle of incidence of the wave on the surface of the structure. Research can be carried out for any materials with known parameters. Also included are modern composite materials, known as metamaterials, characterized by a negative relative permittivity and negative magnetic permeability, and what goes with it negative refractive index [33-44]. This method also allows for testing of materials highly dispersive and observe the phenomenon of electromagnetic wave tunneling in quasi one-dimensional multilayer structure. The second method used is the Finite-Difference Time Domain (FDTD) algorithm [45] allows on the study of electromagnetic wave propagation in the structure built of dielectric materials. The use of Fast Fourier Transform (FFT) allows you to generate a wavelength characteristics, which you can explore the formation and occurrence of band gaps in periodic and aperiodic structures. FDTD algorithm allows the observation of electromagnetic wave propagation in time, and therefore can be observed phenomena occurring inside the multilayer material before transmission through the structure to stabilize.

The Matrix Method transmission is determined by:

$$T = \frac{n_{out}\cos\Theta_{out}}{n_{in}\cos\Theta_{in}}\left|\frac{1}{\Gamma_{11}}\right|^2 \tag{1}$$

where Θ_{out} is the angle at which EMW leaves multilayers, Θ_{in} is angle of incidence of the electromagnetic wave in relation to the normal to the structure, n_{in}, n_{out} are the refractive indexes of the environment surrounding the multilayer system respectively before and after the structure, Γ_{11} - the first diagonal element of the Γ characteristic matrix superlattice described by the relation:

$$\begin{gathered}\Gamma = \frac{1}{t_{in,j+1}}\begin{bmatrix}1 & r_{in,j+1}\\ r_{in,j+1} & 1\end{bmatrix}\cdot\\ \cdot\left[\left[\prod_{j=1}^{J} P_j\right]\frac{1}{t_{j,j+1}}\begin{bmatrix}1 & r_{j,j+1}\\ r_{j,j+1} & 1\end{bmatrix}\right]\\ P_j = \begin{bmatrix} e^{id_j n_j \frac{2\pi}{\lambda}\cos\Theta_j} & 0\\ 0 & e^{-id_j n_j \frac{2\pi}{\lambda}\cos\Theta_j}\end{bmatrix}\end{gathered} \tag{2}$$

where n_j, d_j is respectively refractive index and thickness of the layer j, Θ_j - determined from Snell law angle of incidence of electromagnetic wave in the layer j, λ - the wavelength of incidence wave. The parameters t and s are the Fresnel coefficients which determine the behavior of the electromagnetic wave at the border centers. They depend on the type of polarization and for P-type polarization are:

$$\begin{gathered} t^P_{j,j+1} = \frac{2n_j\cos\Theta_j}{n_j\cos\Theta_{j+1} + n_{j+1}\cos\Theta_j}\\ r^P_{j,j+1} = \frac{n_j\cos\Theta_{j+1} - n_{j+1}\cos\Theta_j}{n_j\cos\Theta_{j+1} + n_{j+1}\cos\Theta_j}\end{gathered} \tag{3}$$

while for S type of polarization they take the form:

$$\begin{gathered} t^S_{j,j+1} = \frac{2n_j\cos\Theta_j}{n_j\cos\Theta_j + n_{j+1}\cos\Theta_{j+1}}\\ r^S_{j,j+1} = \frac{n_j\cos\Theta_j - n_{j+1}\cos\Theta_{j+1}}{n_j\cos\Theta_j + n_{j+1}\cos\Theta_{j+1}}\end{gathered} \tag{4}$$

Use of the algorithm Finite-Difference Time Domain is to change the partial derivatives of the Maxwell equations for the differences relevant to dimension the problem under consideration. In the present case quasi one-dimensional general structure of Maxwell's equations take the form:

$$\begin{gathered}\frac{\partial\vec{D}}{\partial t} = \nabla\times\vec{H}\\ \vec{D} = \varepsilon_0\vec{\varepsilon}_r\cdot\vec{E}\\ \frac{\partial\vec{H}}{\partial t} = -\frac{1}{\mu_0}\nabla\times\vec{E}\end{gathered} \tag{5}$$

where E is the electric field intensity, H magnetic field intensity and D is the electric induction vector. ε_0, μ_0 are electric and magnetic permeability in a vacuum, and ε_r is a relative permittivity of the medium. After using the equations of the normalization.

$$\tilde{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot E,\quad \tilde{D} = \sqrt{\frac{1}{\varepsilon_0\cdot\mu_0}}\cdot D \tag{6}$$

You can transform the system of equations (5) to the quasi one-dimensional system of equations stored in the formalism of the FDTD method.

$$\begin{gathered}\tilde{D}_x\Big|_k^{n+\frac{1}{2}} = \tilde{D}_x\Big|_k^{n-\frac{1}{2}} + \frac{1}{\sqrt{\varepsilon_0\cdot\mu_0}}\cdot\frac{\Delta x}{\Delta t}\left[H_y\Big|_{k+\frac{1}{2}}^{n} - H_y\Big|_{k-\frac{1}{2}}^{n}\right]\\ \tilde{E}_x\Big|_k^{n+\frac{1}{2}} = \frac{1}{\varepsilon_r + \frac{\sigma_r\cdot\Delta t}{\varepsilon_0}}\cdot\left[\tilde{D}_x\Big|_k^{n+\frac{1}{2}} - I_x\Big|_k^{n+\frac{1}{2}}\right]\\ I_x\Big|_k^{n+\frac{1}{2}} = I_x\Big|_k^{n-\frac{1}{2}} + \frac{\varepsilon_r\cdot\Delta t}{\varepsilon_0}\cdot\tilde{E}_x\Big|_k^{n+\frac{1}{2}}\\ H_y\Big|_{k+\frac{1}{2}}^{n+1} = H_y\Big|_{k+\frac{1}{2}}^{n} + \frac{1}{\sqrt{\varepsilon_0\cdot\mu_0}}\cdot\frac{\Delta x}{\Delta t}\left[\tilde{E}_x\Big|_k^{n+\frac{1}{2}} - \tilde{E}_x\Big|_{k+1}^{n+\frac{1}{2}}\right]\end{gathered} \tag{7}$$

In the system of equations (7) I is an auxiliary matrix, n is a step in the one-dimensional k-space, and Δx, Δt describe respectively coordinates discretization of the position and time. In order to ensure the stability of the simulation it is necessary to link the maximum time step with the space discretization factor and speed of light in vacuum c by taking into account the Courant condition.

$$\Delta t = \frac{\Delta x}{2 * c} \tag{8}$$

The simulation used a soft source of electromagnetic waves and Absorbing Boundary Conditions (ABC) in order to provide better quality of results. By using Fast Fourier Transform (FFT), the distribution of wavelength propagating in the simulation space was made.

The paper presents the occurrence of band gaps and transmission properties of the Fibonacci superlattice [46-55]. In order to construct a binary structure describing the spatial distribution of layers, use the following recursive formula:

$$X_0^F = B$$

$$X_1^F = A \tag{8}$$

$$X_{L+1}^F = X_L^F X_{L-1}^F$$

Where L is the number of generations of the superlattice. The first few generations of the layers in the structure shown in Table 1.

Table 1.
Structure of the layers in the Fibonacci superlattice built of materials A and B

Generation (number of layers)	Structure of the layers
0 (1)	B
1 (1)	A
2 (2)	AB
3 (3)	ABA
4 (5)	ABAAB
5 (8)	ABAABABA
6 (13)	ABAABABAABAAB
7 (21)	ABAABABAABAABABAABABA
8 (34)	ABAABABAABAABABAABABAABAABABAA BAAB
9 (55)	ABAABABAABAABABAABABAABAABABAA BAABABAABABAABAABABAABABA
10 (89)	ABAABABAABAABABAABABAABAABABAA BAABABAABABAABAABABAABABAABAAB ABAABAABABAABABAABAABABAABAAB

2. Research

In this study the behavior of the electromagnetic wave propagating in the Fibonacci superlattice. Analysed multilayer consisted of lossless and non-dispersive materials. A material was NaCl n_A = 1.544 or its metamaterial equivalent with a refractive index n_A = - 1.544, while material B was GaAs with n_B = 3.4 [2]. The effect of the refractive index of the ambient material of the multilayer system (n_{in}, n_{out} – Figs. 1-3). Then examined the Fibonacci superlattice transmission in the wavelength range of near-infrared with polarization P and S (Figs. 4-7). In order to investigate the characteristics of wavelength propagating in the superlattice was determined the transmission of electromagnetic wave (Fig. 8) were incidence angle Θ_{in} was equal to 0. Then, using the FDTD algorithm, was examined the propagation of electromagnetic waves in Fibonacci superlattice at a time. The high-band transmission for λ = 450 nm (Figs. 9-11) and a band gap for λ = 520 nm (Figs. 12-14). Simulation parameters are given in the descriptions of figures.

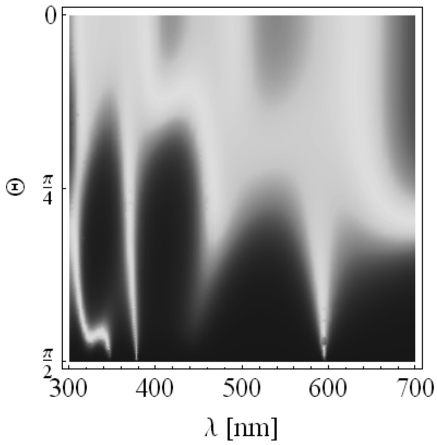

Fig. 1. Transmission map for $n_{in} = n_{out} = 1$, $d_A = d_B = 280$ nm, $n_A = -1.544$, $n_B = 3.4$, $L = 5$ and polarization type S

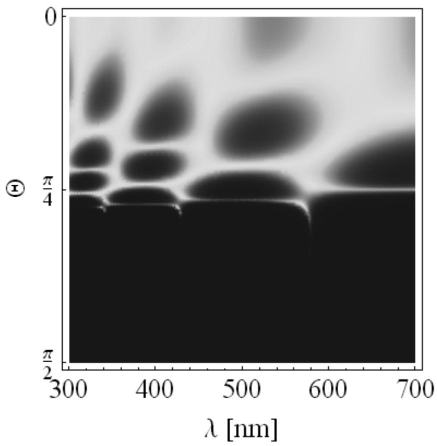

Fig. 2. Transmission map for $n_{in} = n_{out} = 2$, $d_A = d_B = 280$ nm, $n_A = -1.544$, $n_B = 3.4$, $L = 5$ and polarization type S

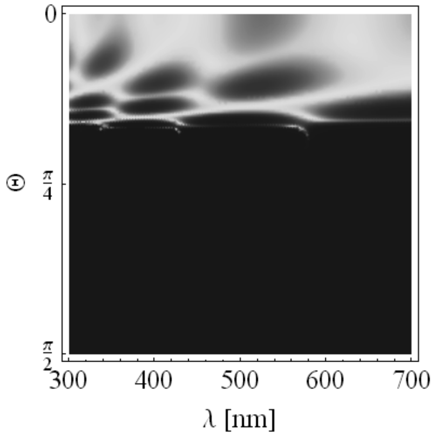

Fig. 3. Transmission map for $n_{in} = n_{out} = 3$, $d_A = d_B = 280$ nm, $n_A = -1.544$, $n_B = 3.4$, $L = 5$ and polarization type S

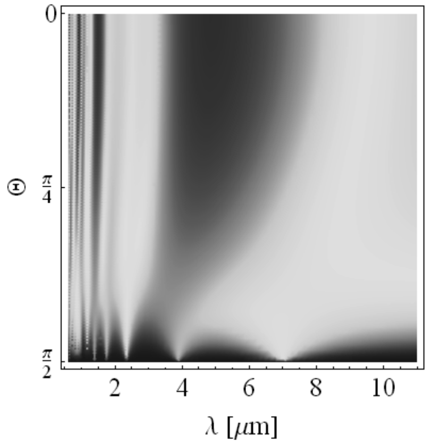

Fig. 4. Transmission map for $d_A = d_B = 350$ nm, $n_A = 1.544$, $n_B = 3.4$, $L = 5$ and polarization type P

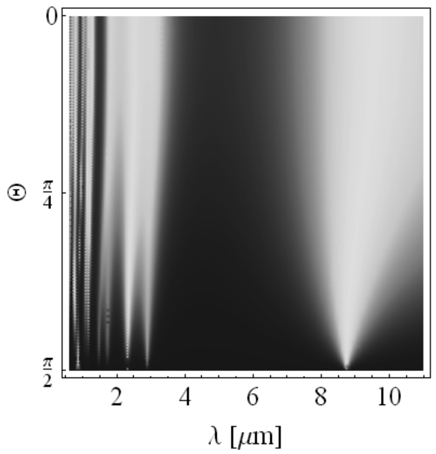

Fig. 5. Transmission map for $d_A = d_B = 350$ nm, $n_A = 1.544$, $n_B = 3.4$, $L = 5$ and polarization type S

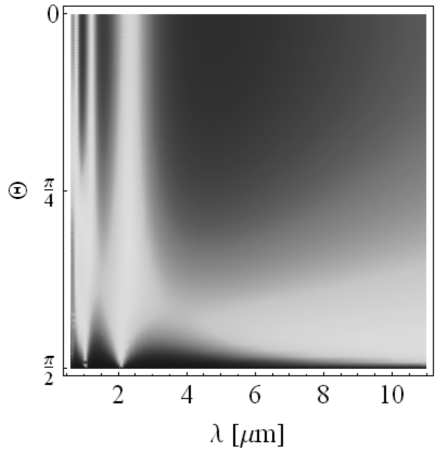

Fig. 6. Transmission map for $d_A = d_B = 350$ nm, $n_A = -1.544$, $n_B = 3.4$, $L = 5$ and polarization type P

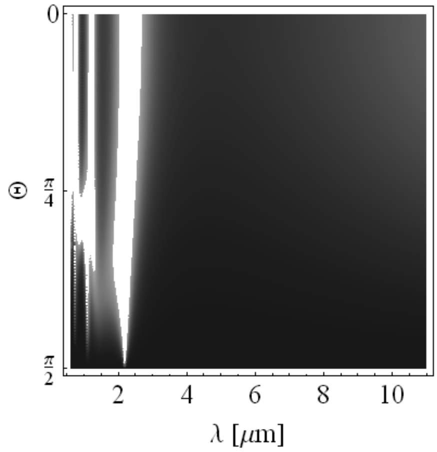

Fig. 7. Transmission map for $d_A = d_B = 350$ nm, $n_A = -1.544$, $n_B = 3.4$, $L = 5$ and polarization type S

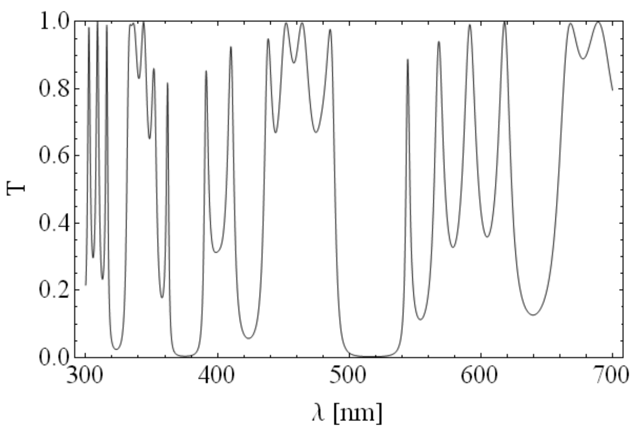

Fig. 8. Transmission for Fibonacci superlattice where $n_A = 1.544$, $n_B = 3.4$, $d_A = d_B = 200$ nm, $\Theta_{in} = 0$, $L = 7$ and polarization type P

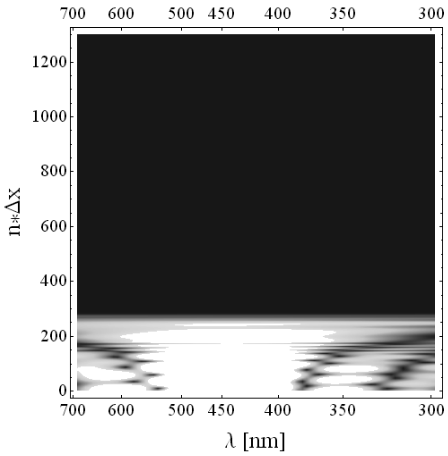

Fig. 9. Wavelength characteristic for λ = 450 nm after T = 1000 timesteps

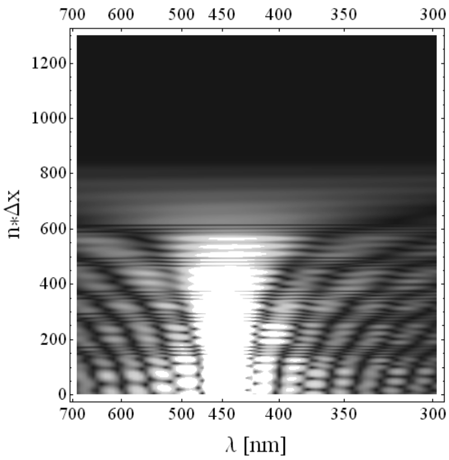

Fig. 10. Wavelength characteristic for λ = 450 nm after T = 3000 timesteps

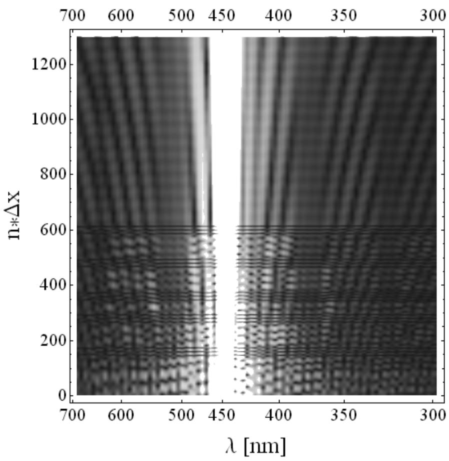

Fig. 11. Wavelength characteristic for λ = 450 nm after T = 8000 timesteps

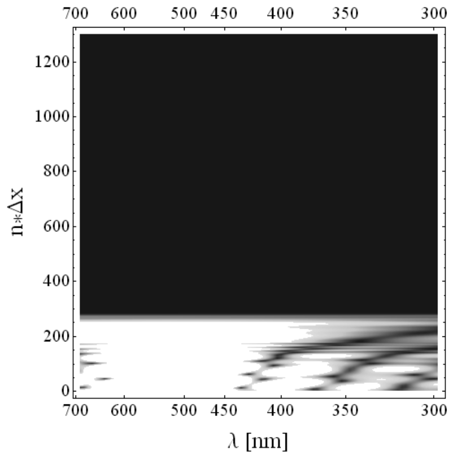

Fig. 12. Wavelength characteristic for λ = 520 nm after T = 1000 timesteps

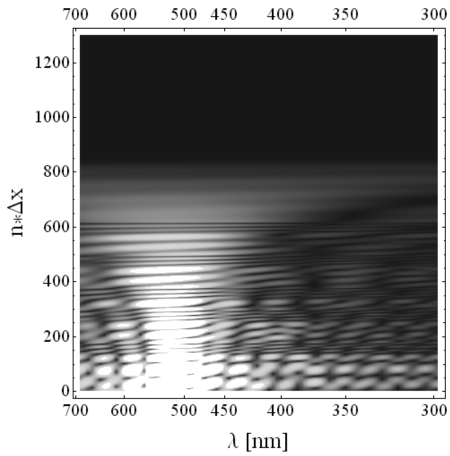

Fig. 13. Wavelength characteristic for λ = 520 nm after T = 3000 timesteps

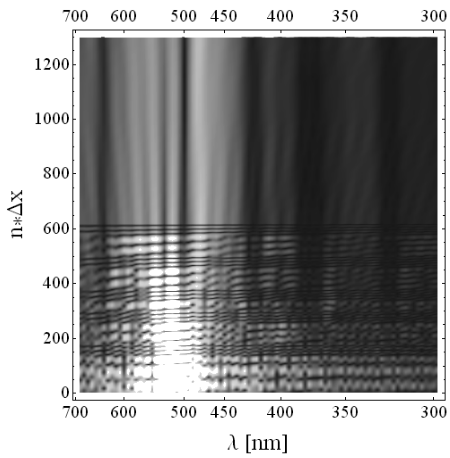

Fig. 14. Wavelength characteristic λ = 520 nm after T = 8000 for timesteps

3. Conclusions

Fibonacci superlattices have been pre-tested in [42]. The purpose of the article was to broaden the knowledge about the behavior of these structures as filters electromagnetic waves with a wavelength range from the near infrared, the effect of the material surrounding the transmission and increasing knowledge of the formation of band gaps. Part of the results of the study are presented in Figures 1-12.

In analysing the impact of the material surrounding the transmission of the Fibonacci superlattice structure observed transmission shift toward smaller angles and the presence above the critical angle a narrow bands of the tunneled electromagnetic wave.

In the near infrared transmission maps are typical and similar to binary systems and Thue-Morse superlattices. Present clear differences depending on the polarization allow the use of superlattices as polarizers for specific ranges of wavelengths and angles of incidence.

Research propagation and distribution of wavelengths using the FDTD algorithm (Figures 9-14) allow you to explore the behavior of the wave in terms of full transmission and the band gaps.

It can be seen that the lighting of the filter with monochromatic light in the wavelength range of the band gap filter at the output causes propagation of low intensity in the range other than the wavelength of the incident beam. This phenomenon can be explained by the formation of standing waves and their interference with the beam inside the structure.

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