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Transmission of polarized light in LHM and RHM materials with periodic refractive index

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ABSTRACT

Purpose: The paper presents the implementation of the Transfer Matrix Method algorithm using Mathematica and the impact of the structure discretization steps number on the filters transmission. Tested materials were made with periodically varying refractive index.

Design/methodology/approach: The properties of the filter transmission made of a materials having a periodically varying refractive index were analyzed. The study used a Transfer Matrix Method algorithm. The materials have a thickness of one micrometer. The refractive index of the analyzed material changed sinusoidally with a wavelength of 500nm. Sinusoid quantizing was performed each for 8, 16, 32, 50 and 60 layers.

Findings: Maps show the nature of the transmission bands. Band structure of RHM materials (positive refractive index) is similar to the structure of the filter constructed of LHM material, characterized by a negative refractive index. Transmission band in a left-handed material has less width at half maximum. Thirty two layers discretization stabilizes the simulations of the tested materials filtration properties.

Research limitations/implications: The paper had not been analyzed for materials with extinction coefficient different from zero. It would be worthwhile to conduct research for materials with variable refractive index of a different nature, for example a triangular or saw tooth shape.

Practical implications: Analysis of the filter materials for a variety of photonic structures allows the prediction and design of materials with specified properties. These tests allow to design filters and mirrors for with very good applications parameters.

Originality/value: The influence of the discretization level of continuous medium with periodically changing material properties on the transmission map stability was analyzed.

Keywords: Left-handed materials; Transmission maps; Discretization; Band structure; Transfer matrix method algorithm

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ANALYSIS AND MODELLING

1. Introduction

Photonic materials are interesting because of their specific transmission properties – the existence of photonic band gap [1-21]. They are used in optics, optoelectronics, solid state physics and photonics. Most systems are constructed as consisting of two or more isotropic components with periodic or aperiodic structure called superlattices or multilayers.

The most common concern were right-handed isotropic homogeneous materials with a small dispersion [17,22,23]. Left-handed materials, with a much larger dispersion, were theoretical curiosity directly resulting from the Maxwell equations [24], until the experimental confirmation of their existence [25-31] in the range of microwave radiation. Work is currently underway on obtaining a left-handed material for visible light [32,33].

To determine the transmission of electromagnetic wave in the multilayer medium transfer matrix method can be used [20,21]. Implementation of this algorithm using Mathematica is presented on the following listing

```
katy[t_, n_, la_] :=
 Transpose[
   NestList[
    {#[[1]] + 1,
      ArcSin[(n[[#[[1]] + 1]][la] / n[[#[[1]] + 2]][la])
        Sin[#[[2]]]]} &, {0, t}, Length[n] - 1]][[2]]

fi[j_, d_, n_, la_, tl_] :=
 2 Pi d[[j]] n[[j]][la] Cos[tl[[j]]] / la

pro[j_, d_, n_, la_, tl_] :=
 {{Exp[ⅈ fi[j, d, n, la, tl]], 0},
  {0, Exp[-ⅈ fi[j, d, n, la, tl]]}}

tp[j_, n_, tl_, la_] :=
 (2 n[[j]][la] Cos[tl[[j]]] / Sign[n[[j]][la]]) /
  (n[[j]][la] Cos[tl[[j + 1]]] / Sign[n[[j]][la]] +
    n[[j + 1]][la] Cos[tl[[j]]] / Sign[n[[j + 1]][la]])

ts[j_, n_, tl_, la_] :=
 (2 n[[j]][la] Cos[tl[[j]]] / Sign[n[[j]][la]]) /
  (n[[j]][la] Cos[tl[[j]]] / Sign[n[[j]][la]] +
    n[[j + 1]][la] Cos[tl[[j + 1]]] / Sign[n[[j + 1]][la]])

rp[j_, n_, tl_, la_] :=
 (n[[j]][la] Cos[tl[[j + 1]]] / Sign[n[[j]][la]] -
    n[[j + 1]][la] Cos[tl[[j]]] / Sign[n[[j + 1]][la]]) /
  (n[[j]][la] Cos[tl[[j + 1]]] / Sign[n[[j]][la]] +
    n[[j + 1]][la] Cos[tl[[j]]] / Sign[n[[j + 1]][la]])

rs[j_, n_, tl_, la_] :=
 (n[[j]][la] Cos[tl[[j]]] / Sign[n[[j]][la]] -
    n[[j + 1]][la] Cos[tl[[j + 1]]] / Sign[n[[j + 1]][la]]) /
  (n[[j]][la] Cos[tl[[j]]] / Sign[n[[j]][la]] +
    n[[j + 1]][la] Cos[tl[[j + 1]]] / Sign[n[[j + 1]][la]])

dj[j_, n_, tl_, polar_, la_] :=
 If[polar, 1 / tp[j, n, tl, la]
   {{1, rp[j, n, tl, la]}, {rp[j, n, tl, la], 1}},
  1 / ts[j, n, tl, la] {{1, rs[j, n, tl, la]}, {rs[j, n, tl, la], 1}}]

gam[d_, n_, la_, tl_, polar_] :=
 Dot[dj[1, n, tl, polar, la],
  Apply[Dot,
   Table[Dot[pro[i, d, n, la, tl], dj[i, n, tl, polar, la]],
    {i, 2, Length[n] - 1}]]]

tran[d_, n_, la_, t_, polar_] :=
 (Last[n][la] Cos[Last[katy[t, n, la]]]) /
   (First[n][la] Cos[First[katy[t, n, la]]])
  (Abs[1 / gam[d, n, la, katy[t, n, la], polar][[1, 1]]]^2)
```

Function *tran[]* returns the transmission coefficient in the range 0-1 for *la* wavelength in nm, *t* – incident angle to the surface normal of the superlattice. List *n* describes the refractive indices of the superlattice layers as a function including the dispersion, and *d* is corresponding to the thickness of the layers in nm. Boolean variable *polar* describes the polarization of the incident electromagnetic wave. For P-type polarization variable has a value of *True*, and *False* corresponds to the S-type of polarization [20].

Function *gam[]* designates the characteristic matrix of the multilayer system in which electromagnetic wave propagation in the dielectric layer is determined by a matrix function described by *pro[]*. The angles of incidence are described in the layers by the variable *tl*, and designated by the function *katy[]* from Snell's law.

Transmission of electromagnetic waves on the boundaries is determined by the array returned by the function *dj[]*. Depending on the type of polarization shall be selected corresponding reflectance and transmittance Fresnel amplitude coefficients. Frasnel amplitude transmission coefficient describes the function *tp[]*, and reflectance coefficient *rp[];* in the case of S-type polarization there are respectively functions *ts[]* and *rs[]*.

2. Research

In this study material having a thickness of 1000 nm. The refractive index of the analyzed material changed sinusoidally with a wavelength of 500 nm. Sinusoid quantizing was performed each for 8, 16, 32, 50 and 60 layers for each of the cases considered (method of digitization in the first case is shown in Table 1). In the first case refractive index has values in the range 1-3 – right-handed material. The second is in the range (-3)-(-1) – left-handed material. Multilayers were placed in the air.

Table 1.
Discretization of the sine function by spline function composed of equal thickness layers. The horizontal axis is the sample thickness in nm, and the vertical is the refractive index

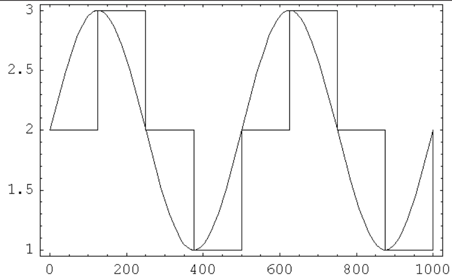
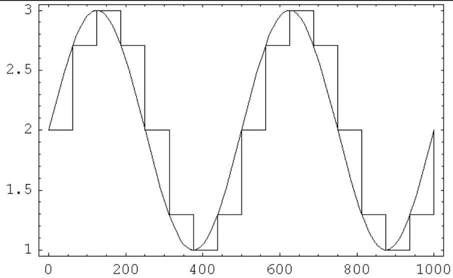
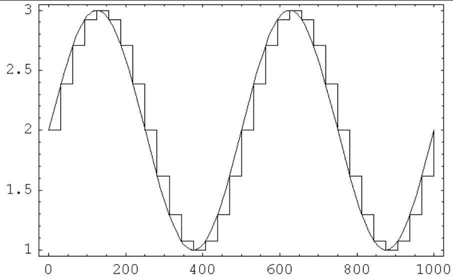
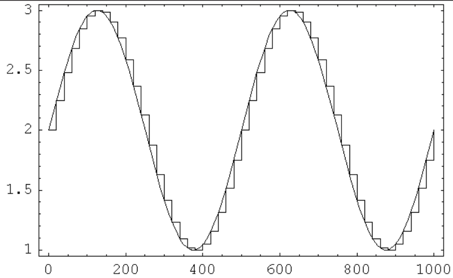
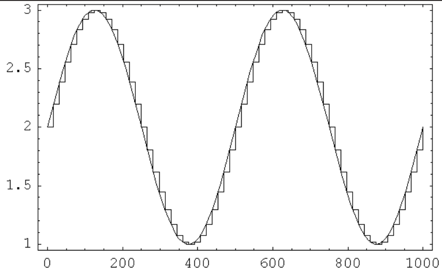
Number of layers	Function diagram sin () and spline function
8	
16	
32	
50	
64	

Table 2.
Multilayers transmission maps with refractive index in the range 1 to 3, for different levels of discretization (number of layers) and different polarity. The horizontal axis is the wavelength in nm, and on the vertical is monochromatic electromagnetic wave angle of incidence to the normal to the sample surface

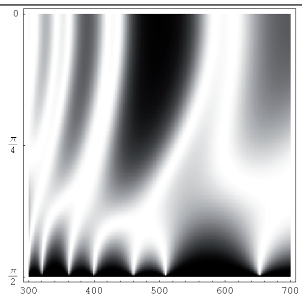
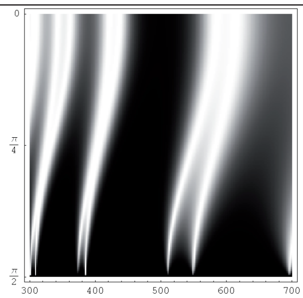
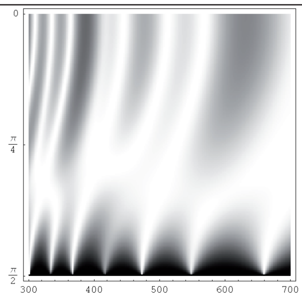
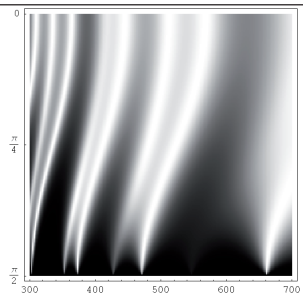
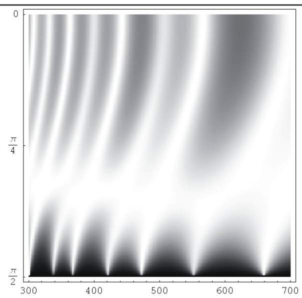
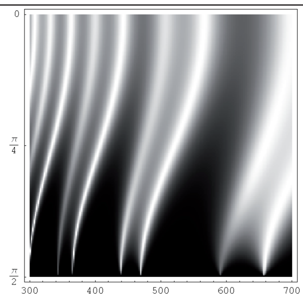
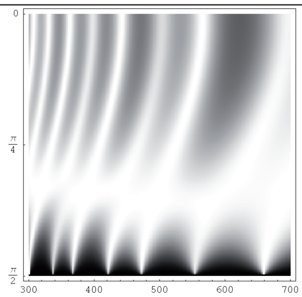
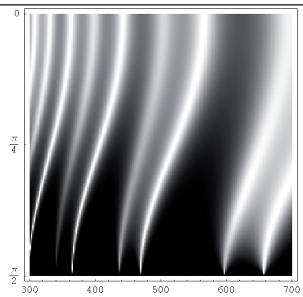
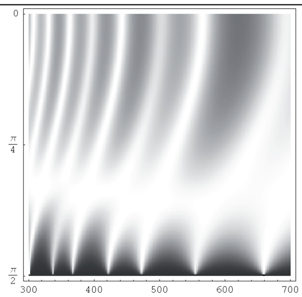
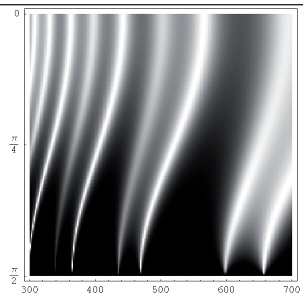
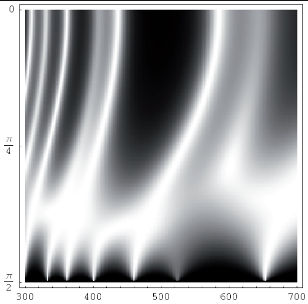
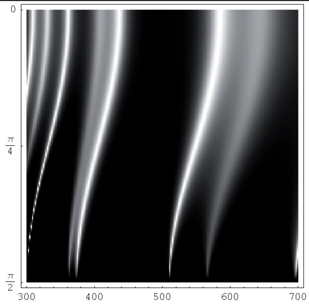
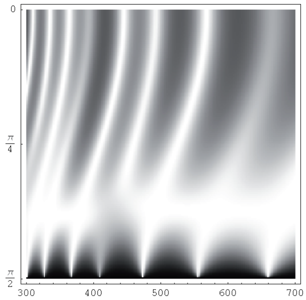
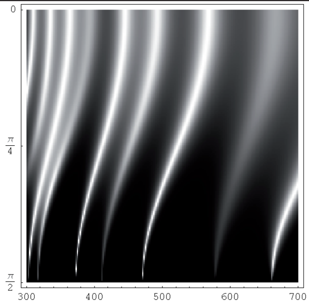
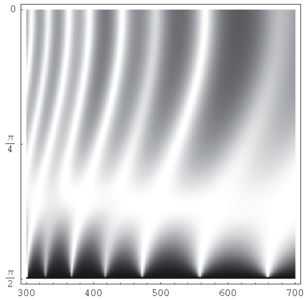
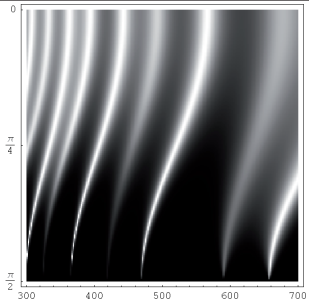
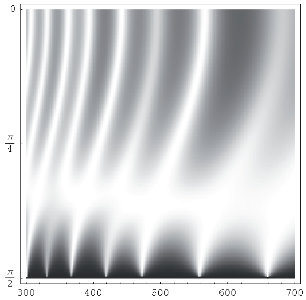
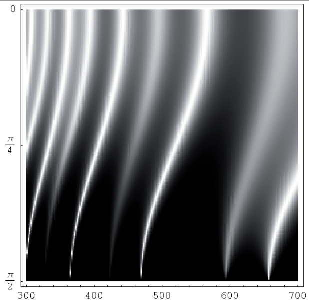
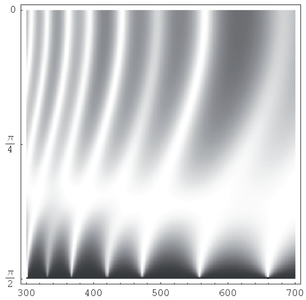
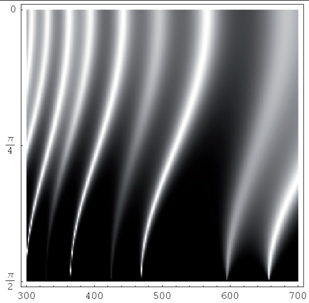
Number of layers	P-type polarization	S-type polarization
8		
16		
32		
50		
64		

Table 3.
Transmission maps of multilayer systems with a refractive indicesin the range (-1)-(-3)

Number of layers	P-type polarization	S-type polarization
8		
16		
32		
50		
64		

Tables 2-3 shows the superlattice transmission maps. The horizontal axis represents the length of the electromagnetic wave in the range 300 nm-700 nm, while the vertical axis is the angle between the incident wave and the normal to the plane of the multilayer system (0-½π). White color determines a full transmission of the incident wave, and black color its absence.

3. Conclusions

The studied cases (Tables 2 and 3) show the characteristics band structure specific for periodic multilayers. It is also visible a decrease width at half maximum of bands in the left-handed multilayers. Structure in both cases were similar.

It should be noted the discrepancy between the plots with 8 layers and the others in which the state with increasing the number of layers is stabilizing. Quantization consisting of thirty two layers is sufficient to stabilize the simulation of material with periodically variable refractive index.

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