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Elastic modules identification by layered composite beams testing

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ABSTRACT

Purpose: The study aims to predict elastic properties of composite laminated plates from the measured mechanical properties.

Design/methodology/approach: Elastic constants of laminates and damping properties have been determined by using an identification procedure based on experiment design, and multi-level theoretical approach.

Findings: The present paper is the first attempt at proposing a novel adaptive procedure to derive stiffness parameters from forced sandwich plate's vibration experiments.

Research limitations/implications: In the future the extension of the present approach to sandwich plates with different core materials will be performed in order to test various experimental conditions.

Practical implications: Structures composed of laminated materials are among the most important structures used in modern engineering and especially in the aerospace industry. Such lightweight and highly reinforced structures are also being increasingly used in civil, mechanical and transportation engineering applications.

Originality/value: The main advantage of the present method is that it does not rely on strong assumptions on the model of the plate. The key feature is that the raw models can be applied at different vibration conditions of the plate by a suitable analytical ore approximation method

Keywords: Computational material science; Composite materials; Laminated plates; Elastic constants; Identification

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ANALYSIS AND MODELLING

1. Introduction

Since the late 1950's many papers have been published on the vibration of sandwich structures [1-9]. All the models discussed so far are based on the following assumptions: (a) the viscoelastic layer undergoes only shear deformation and hence the extensional energy of the core is neglected; (b) the face sheets are elastic and isotropic and their contribution to the shear energy is neglected; and (c) in the face-sheets plane sections remain plane and normal to the deformed centerlines of the face-sheets. However, as the frequency increases the results calculated from these models disagree strongly with measurements.

The simple laminate theories are most often incapable of determining the 3-D stress field in the lamina. Thus, the analysis of composite laminates may require the use of laminate independent theory or a 3-D elasticity theory. Exact three-dimensional solutions [7,8] have shown the fundamental role played by the continuity conditions for the displacements and the transverse stress components at the interfaces between two adjacent layers for an accurate analysis of multilayered composite thick plates. Review and assessment of various theories for modeling sandwich composites may be found in [10,11]. Further, these elasticity solutions demonstrated that the transverse normal stress plays a predominant role in these analyses. However, accurate solutions based on the three-dimensional elasticity theory are often intractable.

The purpose of our study is the elaboration of a stable identification algorithm allowing one to uniquely determine the elastic modules, including the transverse ones. The problem of identification necessarily includes the planning of experiments, the construction of a calculation model, and the identification schemes themselves. In this study, the construction of the model is considered in two plans: first, adaptation of the model to the kind of specimens and experimental equipment employed and, second, adaptation of the model to the identification scheme. The reliability of the results obtained was estimated by analyzing the robustness of the calculation schemes suggested (Tab. 1).

Table 1.
Calculation scheme

verification of the model	verification of the identification scheme	identification	analysis of reliability (postanalysis)

The verification of the model was considered. New combined criteria of identification — schemes averaged over the calculation results for a homogeneous beam and for a sandwich with a core identical to the homogeneous beam and rigid outer layers are considered. The error function is chosen as the sum of error functions for the homogeneous beam, and for the sandwich. In the present study, combined identification schemes making it possible to unequivocally determine the transverse modules and Poisson ratio are suggested. Based on the classical scheme of three-point bending of a beam and refined calculation schemes, a method is elaborated for determining the elasticity characteristics of a homogeneous beam and of the face layers and core of a sandwich beam by minimizing the deviations of experimental deflections of the beams from calculated ones. It is shown that, for an adequate identification of the characteristics on the basis of refined models, it suffices to use second-order approximations across the thickness of layers of sandwich beams and third-order for beams with the soft coverage. The elastic modules were also determined from measured vibration eigen-frequencies of the beams.

The identification of mechanical characteristics of thin-walled layered elements is a rather complicated problem. Many studies are dedicated to this problem, both theoretical, with a mathematical substantiation of algorithms, and experimental, where one or several parameters are determined. An adequate description of the mechanical properties of layered thin-walled elements is based on the construction of a refined theoretical model [12-18] and on the choice of a testing scheme making it possible to uniquely determine some modulus or a group of moduli.

The method of static experiments used in [19] allows one to separately find the shear modulus of an orthotropic composite from tension tests at a certain angle to its symmetry axes. In [20], a more general approach with a specific choice of the strain field is suggested, which undoes the contribution of all components of the strain tensor except that needed for determination of a certain modulus. This enables one to theoretically determine each elastic modulus separately. But there still remains the question of practical realization of such strain fields in actual experiments. Experimental schemes for determining the elastic moduli by using plates eigen-frequencies are known [21,22]. A parametric identification of vibrating systems is the process of finding mathematical and parameterised models for system, which is based on measured excitation and/or response signals is proposed in [23]. In normal cases, the excitation is the force, the response signal – the vibration displacement, velocity or acceleration. However, they are unsuitable for determining the elastic modulus along the normal to a plate. In [24-26], an adaptive algorithm on the basis of generalized kinematic approximations and the classical Galerkin method for an elliptic system of equations of the linear theory of elasticity is advanced. The elastic moduli are identified by comparing the theoretically determined eigen-frequencies with experimental data. Contrary to the present study, these investigations do not pay sufficient attention to the determination of transverse moduli.

2. Some aspects of beam modeling

Various high-order displacement models have been developed in the literature by considering combinations of displacement fields for in-plane and transverse displacements inside a mathematical sub-layer. In order to obtain more accurate results for the local responses, another class of laminate theories, commonly named as the layer-wise theories, approximate the kinematics of individual layers rather than a total laminate using the 2-D theories [24-26]. These models have been used to investigate the phenomena of wave propagation as well as vibrations in laminated composite plates. Numerical evaluations obtained for wave propagation and vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the efficient displacement field for economic analysis of wave propagation and vibration. The numerical method developed in this paper follows a semi-analytical approach with an analytical field applied in the longitudinal direction and a layer-wise displacement field employed in the transverse direction. The goal of the present paper is to develop a simple numerical technique, which can produce very accurate results compared with the available analytical solution. The goal is also to provide one with the ability to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis.

Let us consider now such kinematic assumptions ($U=U_e+U_d$) for a symmetrical three-layered plate of thickness $2H_p$ (only cylindrical bending is considered):

$$U_e - \begin{cases} u=\sum_{i,k} u_{ik}{}^e z^{2i-1}\varphi_k(x), & 0<z<H,\\ w=\sum_{i,k} w_{ik}{}^e z^{2i-2}\gamma_k(x), & 0<x<L, \end{cases}$$

$$U_d - \begin{cases} u=\sum_{i,k} u_{ik}{}^d (z-H)^i \varphi_k(x), & H<z<H_p,\\ w=\sum_{i,k} w_{ik}{}^d (z-H)^i \gamma_k(x) & 0<x<L. \end{cases} \quad (1)$$

Here $\varphi_k(x)$, $\gamma_k(x)$ – are apriory known coordinate functions (for every beam clamp conditions), $u_{ik}{}^e$, $w_{ik}{}^e$, $u_{ik}{}^e$, $w_{ik}{}^e$ – unknown set of parameters.

By substituting Eqs. (1)) into the following variation equation

$$\int_{t_1}^{t_2}\int_V (\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{zz}\delta\varepsilon_{zz}+\tau_{xz}\delta\varepsilon_{xz}-\rho\frac{\partial u}{\partial t}\delta\frac{\partial u}{\partial t}-\rho\frac{\partial w}{\partial t}\delta\frac{\partial w}{\partial t})dVdt=\int_S P\delta U, \quad (2)$$

and also assuming single frequency vibration
($u_{ik}^e=\overline{u}_{ik}^e e^{i\omega t}$, $w_{ik}^e=\overline{w}_{ik}^e e^{i\omega t}$, $u_{ik}^d=\overline{u}_{ik}^d e^{i\omega t}$, $w_{ik}^d=\overline{w}_{ik}^d e^{i\omega t}$)
we obtain the set of linear algebraic equations for the amplitudes [24-26]:

$$[A]\overline{U}=\begin{bmatrix} A_1 & A_d \\ A_d^T & A_2 \end{bmatrix}\begin{bmatrix} \overline{U}_e \\ \overline{U}_d \end{bmatrix}=f \quad (3)$$

For a greater number of lamina this equation has the following form for each additional layer

$$U_d^n - \begin{cases} u=\sum_{i,k} u_{ik}\left(z-H^{(n)}\right)^i \varphi_k(x), & H^{(n)}<z<H^{(n+1)},\\ & 0<x<L,\\ w=\sum_{i,k} w_{ik}{}^n\left(z-H^{(n)}\right)^i \gamma_k(x), & n=1,\ldots,N, \end{cases} \quad (4)$$

Here $H^{(n)}$ are the low bounds of the n-th layer, respectively. Matrix $[A]$ are found by double integration through the thickness and along the length of the beam. Note that, $N=1$ and $N=2$ represent the cases of symmetrical three- and five-layered plates, respectively.

The corresponding frequency equation for the material with the viscous damping should be written such

$$-\omega^2[M]\overline{U}+i\omega[C]\overline{U}+[K]\overline{U}=[A]\overline{U}=\overline{f} \quad (5)$$

This is the traditional frequency domain method which is normally used in linear elastic system investigations.

These theories are flexible and its demonstrate good convergence results for various layer thickness, number and for different mechanical properties of layers. In most cases it is enough the 3-4 terms in approximations (1) thru the thickness of layers [24-26].

3. Experimental investigations

Beams made of plastic foams of trademarks 3715 and 6718 (General Plastic, USA) and sandwich beams with CFRP load-carrying layers and a core made of the plastic foam of trademark 3715 were tested in three-point bending (see Fig. 1). The thickness of CFRP layers, made of a plain-weave fabric, was 0.5 mm, and the fibers were oriented along and across the layers. The three-layer beam had a rectangular cross section, length 0.6 m, width 0.028 m, and height 0.0264 m. The apparent density of the plastic foams of trademarks 3715 and 6718 was 240 and 277 kg/m3, respectively, and the weight per unit length of the sandwich was 0.207 kg/m. The beams were loaded stepwise. The maximum deflections were measured by dial gages with a scale-division value of 0.01 mm. The radii of cylindrical supports and the width of the loading indenter were 5 mm.

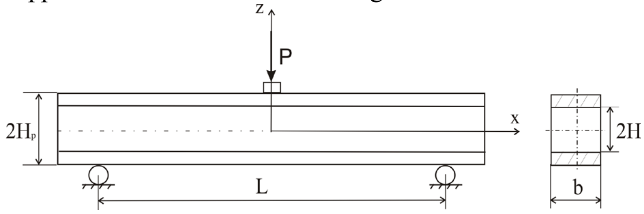

Fig. 1. Loading scheme (a) and cross section (b) of a three-layer beam

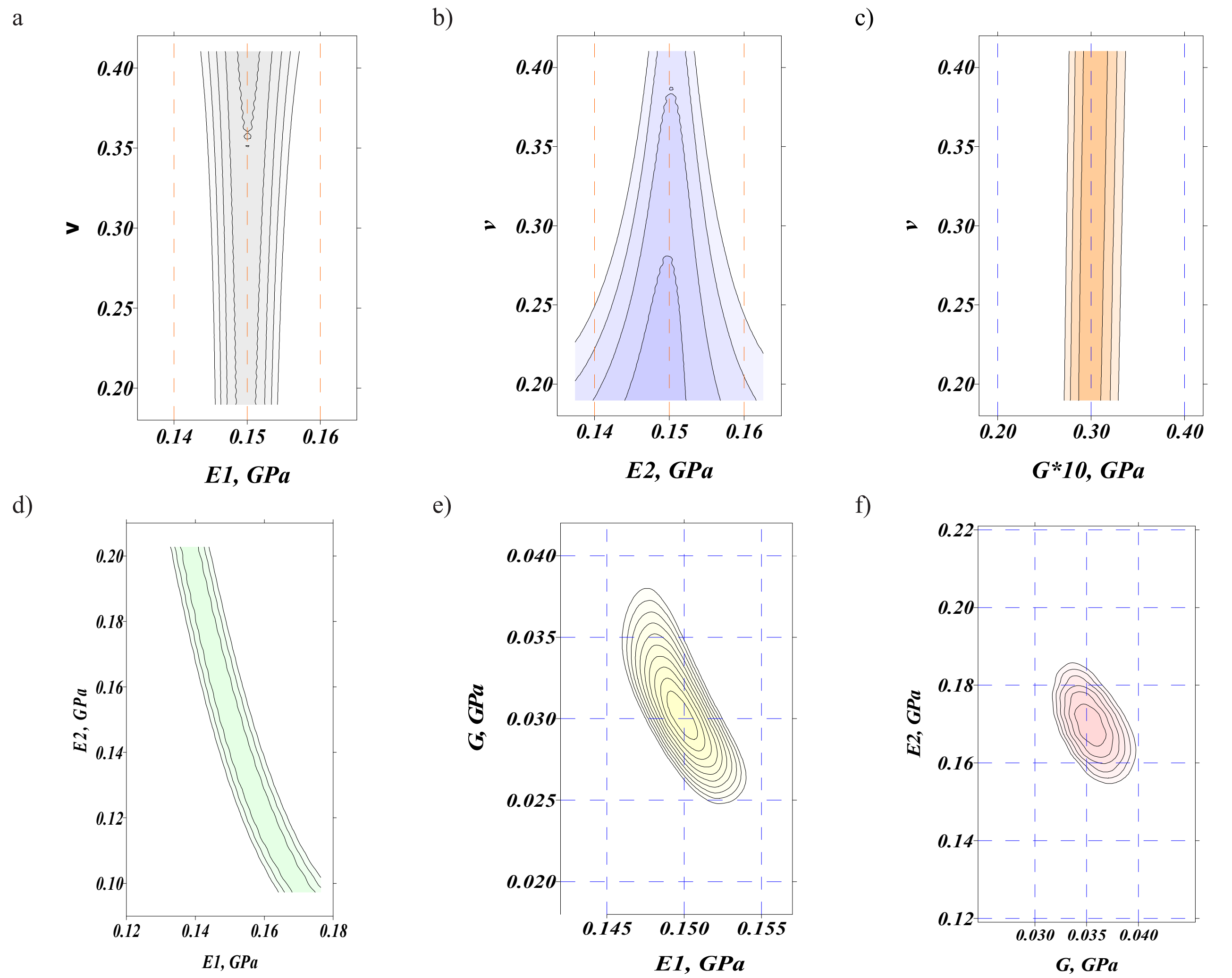

Fig. 2. $E_1-\nu$ (a), $E_2-\nu$ (b), $G-\nu$ (c), E_1-E_2 (d), $E_1,-G$ (e) and $G-E_2$ (f) maps (in statics)

The experiment set up for dynamic investigation includes: a B&K Pulse System (Sound & Vibration Multi-analyzer), laser vibrometer, sample, shaker and power amplifier. These results are kindly given by prof. M. Crocker on the basis of works of his graduate students

4. Verification of identification

The identification of elastic moduli by the results of static tests is realized by the method of minimization of the error function (6):

$$\Delta=\sum_{i}^{N_P}\frac{\left|w_i^{\exp}-w_i\right|}{w_i^{\exp}}, \qquad (6)$$

which is the relative difference, averaged over all loads, between the deflections and determined experimentally and theoretically on the basis of refined model (1-5), respectively.

To verify the identification scheme used, we will consider additionally several maps of error function which include the transverse modulus and Poisson ratio. The error function is defined as the average of differences between the calculated static deflections w_i i of a homogeneous beam in specified ranges of E_2 and ν and the deflections $w_i^{(0)}$ calculated with average values $E_{10}, E_{20}, G_0, \nu_0$ and ν_0 (6).

Fig. 2 depicts some maps of the error function Δ_0 in pairs with the transverse moduli. Similar results were also obtained in dynamic tests.

a)

b)

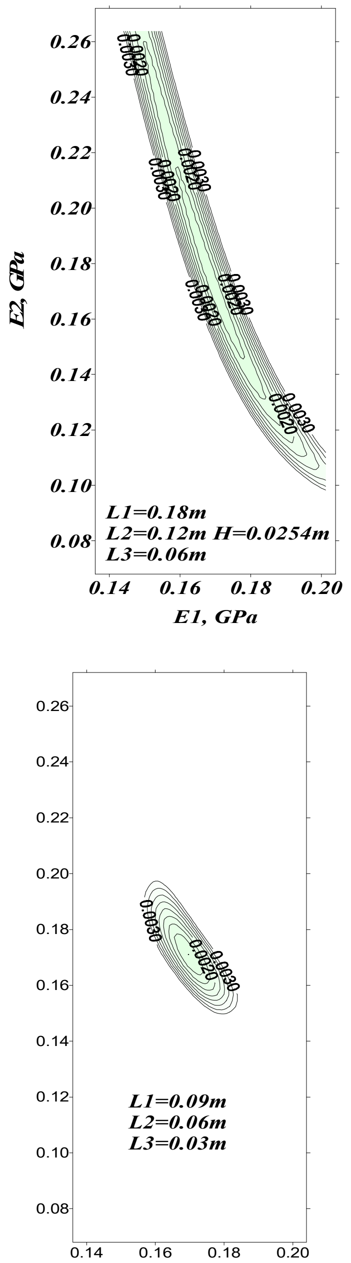

Fig. 3. Map for parameters E_1, E_2, beam thickness *H=0.0254* m: (a) -length *L=0.18, 0.12, 0.06* m; (b) -length *L=0.09, 0.06, 0.03* m

As seen from data of the Fig. 2, in the identification scheme used, the transverse modulus and coefficient cannot be determined unequivocally. Theoretically, they can be found from static tests by reducing the minimum length of beam span to two or three thicknesses of the beam.

Let us consider the three-point test by simultaneously use of beams of various lengths *L, 2L/3, L/3* (Fig.3 - beam thickness *H=0.0254* m). The difference function is:

$$\Delta = \left(w_0(0) - w(0)\right)\Big|_{L=L_0/3} + \left(w_0(0) - w(0)\right)\Big|_{L=2L_0/3}\Big/8 + \left(w_0(0) - w(0)\right)\Big|_{L=L_0}\Big/27$$

But, at such spans, which are practically comparable with beam thickness, the local deflections in supports

must be taken into account. In dynamics, a considerable improvement in identification of the moduli is possible only in a range of sufficiently high frequencies. Such high-frequency measurements are rather complicated too. The difficulties in determining the transverse moduli can be explained if the vibration forms of the beam are examined (Fig. 4). One can notice that only at a frequency of about 10-15 kHz does the vibration wavelength approach the minimum length of a beam in static tests at which it is possible to separate the moduli E_1 and E_2.

In Fig. 3 little influence on mechanical properties of beam can be noticed, that as for dynamic so for static tests some modules. For dynamic tests only shorter beams or high frequency numerical experiments are informing (Fig. 3b). For static tests only shorter beams give the variance in the difference function. It serves as basis for the following algorithm of identification of the modules: identification of the longitudinal module E_1 on the basis of testing enough

long and thin beams and determination of transverse modules on the basis of testing of short beams or use of information about eigen-frequencies of higher ran

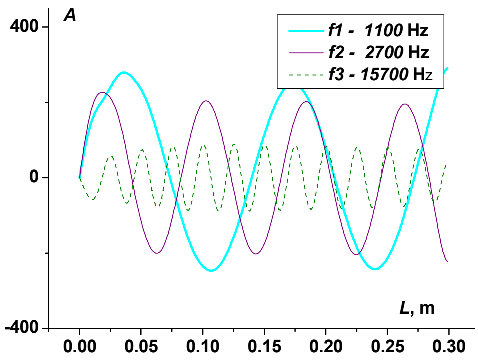

Fig. 4. Vibration modes of a homogeneous beam at $f1 = 1.1$ (1), $f2 = 2.7$ (2), and $f3 = 15.7$ (3) kHz

5. Combined identification

Let us consider new combined criteria of identification - schemes averaged over the calculation results for a homogeneous beam and for a sandwich with a core identical to the homogeneous beam and rigid outer layers. The error function is chosen as the sum of error functions for the homogeneous beam, Δ_C, and for the sandwich, Δ_S, in static tests

$$\Delta=\Delta_c+\Delta_s=\alpha\sum_i^{N_f}\left(\frac{\left|w_i^{\exp,c}-w_i^c\right|}{w_i^{\exp}}\right)+\beta\sum_i^{N_f}\left(\frac{\left|w_i^{\exp,s}-w_i^s\right|}{w_i^{\exp,s}}\right) \qquad (7)$$

or as the sum of dynamic discrepancies for the homogeneous beam, Δ_{DC}, and the sandwich, Δ_{Df}, namely

$$\Delta_D=\Delta_{Dc}+\Delta_{Df}=\alpha\sum_i^{N_f}\left(\frac{\left|f_i^{\exp}-f_i^c\right|}{f_i^{\exp}}\right)^2+\beta\sum_i^{N_f}\left(\frac{\left|f_i^{\exp s}-f_i^{c,s}\right|}{f_i^{\exp s}}\right)^2 \qquad (8)$$

Here, α and β are the weight factors. They were determined by numerical experiments during the minimization process and roughly corresponded to an equal contribution of both summands. Combined static-dynamic criteria, for example, $\Delta_S = \Delta \quad + \Delta_D$ can also be employed.

In [22], it is proved that function (6) is convex.

However, we should note that, for some parameters, its level lines are excessively extended (see Fig. 3a-d). In the case of application of the combined scheme of identification, they become more convex (see Fig. 5).

Let us consider the same maps of parameters for a sandwich in which the mechanical characteristics of core are the same as those of a homogeneous beam. Here, the $E_2-\nu$ and $G-\nu$ maps for the inner layer are still nonconvex, but the E_1-E_2 maps for the core and the $\nu-G_f$, $\nu-E_{xz}^{f}$, and $\nu-E_{zz}^{f}$ maps become convex (Fig. 5).

As seen from the data of Fig. 5, additional information can be used for an unequivocal determination of the transverse modulus E_2 and Poisson ratio ν of the core and, simultaneously, of the moduli of outer layers. For this, it is sufficient that the equal-value level lines of the error function be convex and contract to a point as the error decreases. If, by some criteria, the maps (level lines) are striplines (see Fig. 2a-d), it is necessary to search for variants where, with other schemes of experiments, they become convex. In this connection, it is expedient to consider combined criteria of identification that take into account this information.

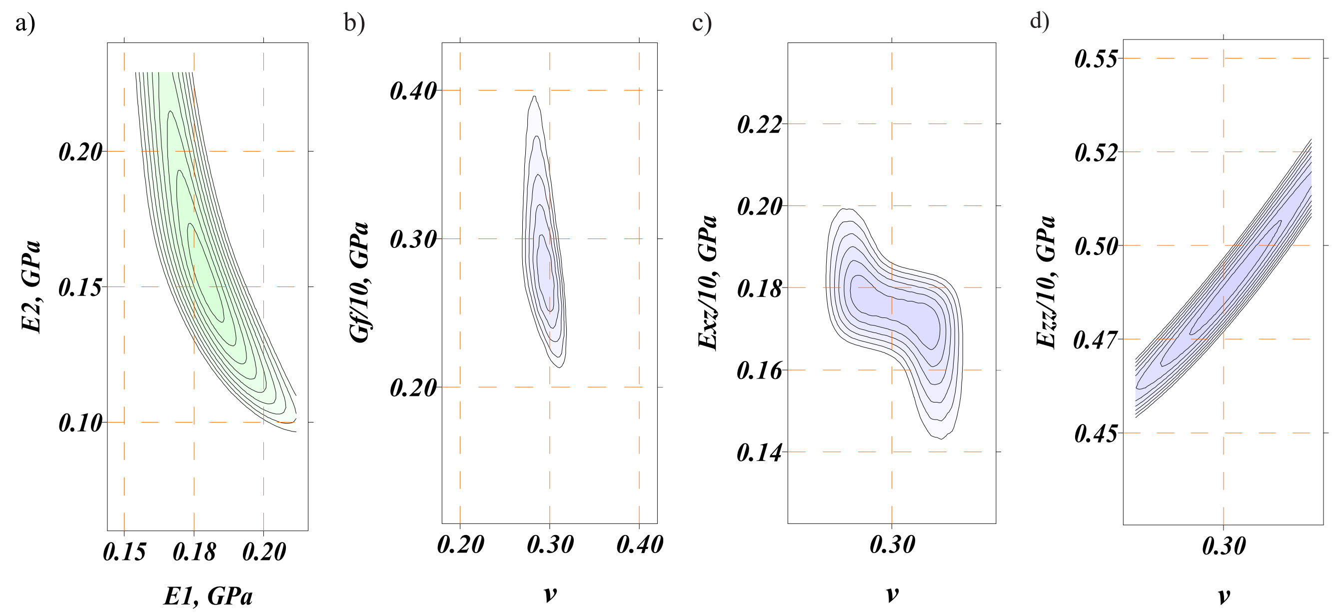

Fig. 5. E_1-E_2 (a), $\nu-G_f$ (b), $\nu-E_{xz}^{f}$ (c), and $\nu-E_{zz}^{f}$ (d) maps (sandwich in statics)

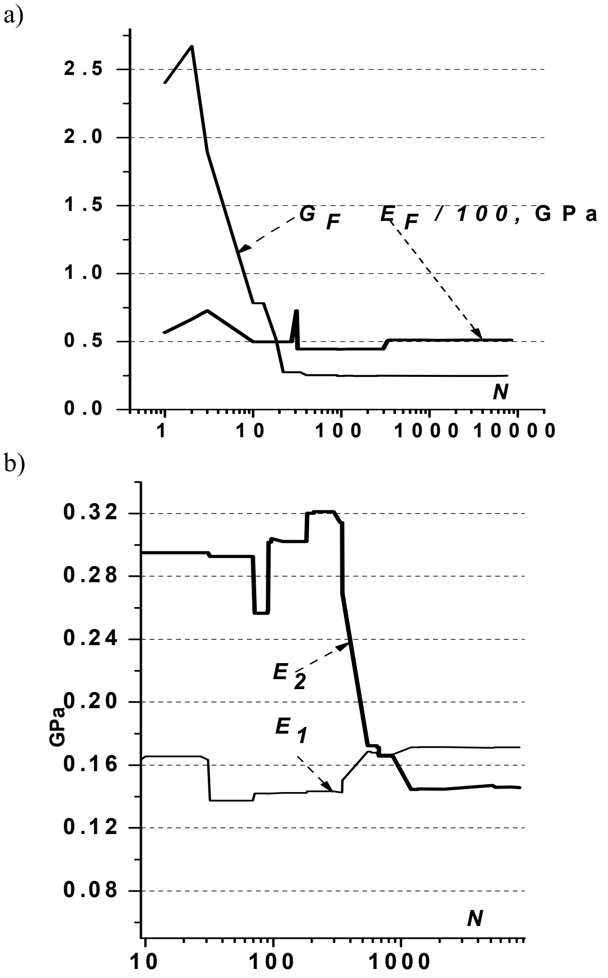

Fig. 6. Stepwise identification of the moduli E_f (a), E_{1c}, E_{2c} (b), and G_c (c) in the combined scheme. C_F - error function

Let us analyze the identification of the moduli in the case of simultaneous use of data on a homogeneous beam and a beam with a core (material 3715). The error function is taken in form (7), with the moduli E_f, G_c, E_{1c} and E_{2c} as the parameters to be optimized. The other moduli of the sandwich are assigned approximately because of their weak influence on the dynamic and static properties of layered plates. Fig. 6 illustrates the identification processes.

Let us consider five variable parameters in the combined identification scheme, E_f, G_c, E_{1c}, E_{2c}, and ν. Here, we will additionally consider the Poisson ratio ν of the homogeneous beam. The processes of identification of the moduli according to different schemes are illustrated in Fig. 7.

We should note that, in the combined identification scheme, the Young's modulus E_2 and Poisson ratio ν are also determined. The final results of the process of identification of the moduli in the different schemes are enclosed in ovals. As seen from data [26], the identified Young's E_1 and shear G moduli for materials 3715 and 6718 coincide, accurate to 1.5%, with data of the General Plastic company (USA), while the Young's modulus E_2 differs by 6-11.5%. The accuracy of agreement with the modulus E_2 determined with the help of combined criterion (7) in bending of a three-layer beam makes 1%.

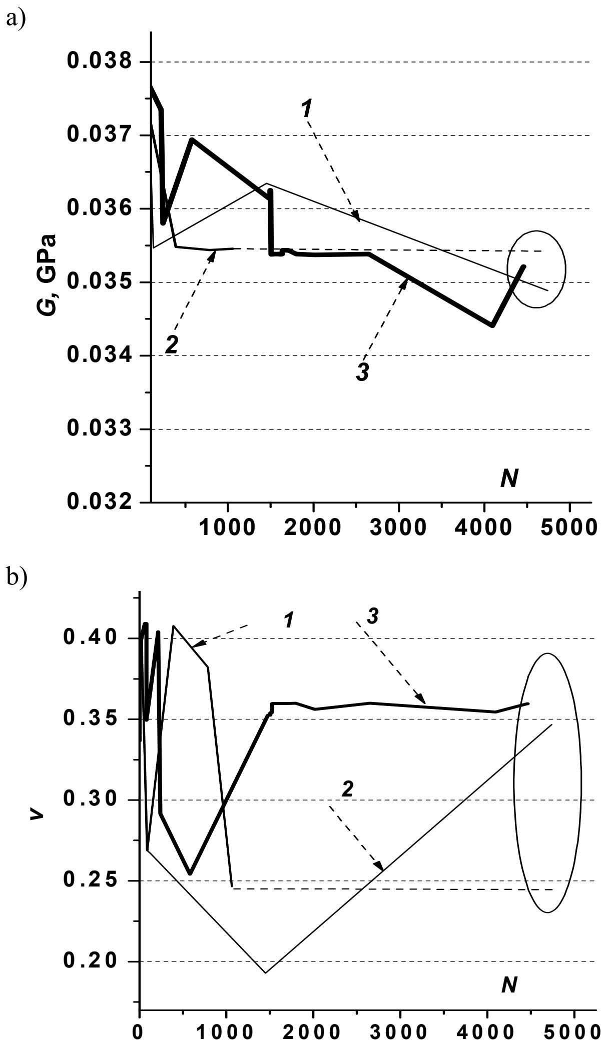

Fig. 7. Stepwise identification of the parameters G_c (a) and u (b) in the combined scheme: 1 - homogeneous beam, 2 - sandwich beam, and 3 - combined scheme

6. Conclusions

In the present study, theoretical models for investigations into the vibrations and damping of layered composite plates are developed. A rational approximation of the field of displacements is established, which al lows one, at a small number of parameters, to predict the dynamic behavior of a beam. A new procedure for determining the parameters of the dynamic rigidity of three-layer plates is suggested, which was used to find the equivalent values of elastic moduli for a Timoshenko beam. We should note that the method presented does not require rigorous assumptions concerning the plate model. In this study, based on the multilevel theoretical approach described and a combined procedure of identification for some composite plates, their elastic moduli will be determined.

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