Journal of Achievements in Materials and Manufacturing Engineering
International Scientific Journal published monthly by the World Academy of Materials and Manufacturing Engineering

Numerical models of a valve system used in railway hydraulic dampers

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ABSTRACT

Purpose: The aim of this paper is to develop and validate a linear and nonlinear numerical 2/3D models of the spring washer stack and a system model of an entire hydraulic damper.

Design/methodology/approach: Three types of numerical models are investigated. Linear and non-linear 2D models developed in Matlab program, and 3D nonlinear model developed in Ansys software.

Findings: The system model of entire hydraulic damper and detail numerical 2/3 D model of the spring washers stack including the boundary conditions for simplified and advanced analysis were developed.

Research limitations/implications: It is important to provide a model functionality allowing for calculation of spring washer stacks groups having the opening limiter. Spring washer stack stress and opening characteristics vs. applied pressure are determined with simplified analytically derived model and full 2D model including almost all significant forces and moments in a stack of circular plates. An advantage of a simplified spring washer stack model is possibility of its rapid engineering calculations, e.g. performed in Matlab.

Practical implications: The valve model allows to determine the critical von Misses stress level and fatigue critical limit in elastic components of a valve system. Damper force and valve durability expressed in life-cycles are the optimization criteria considered during selection and tuning of a valve system.

Originality/value: A new valve system was developed in two versions, i.e. simplified and advanced. The model allows durability prediction at the design stage reducing the testing costs of low-performance valve systems.

Keywords: Hydraulic damper; Damping force; Finite Elements Method; Valve system

Reference to this paper should be given in the following way:

M. Woźniak, Numerical models of a valve system used in railway hydraulic dampers, Journal of Achievements in Materials and Manufacturing Engineering 73/2 (2015) 190-198.

ANALYSIS AND MODELLING

1. Introduction

Suspension systems of high speed trains are strongly subjected to railway roughness which excites structural vibrations. The vibrations are passed from railways to the bogie and further to the train body and their components. In turn, they negatively affect the train stability and passengers' comfort [1]. New methods, to reduce high-speed vibrations such as active suspension modules, are continuously developed; nevertheless passive systems are still commonly in use due to their standard design, better reliability and lower costs. The key suspension component is a hydraulic damper [2] which significantly influences the passenger comfort and train stability through chosen damping characteristic of a hydraulic damper.

As has been proved in [3 - 5] a numerical method are a good approach to select an optimal spring washer setting starting with a numerical computation instead a workshop activities and experimental testing.

To determine the optimal parameters of the valve system can be used simplified and advanced models.

The application of a numerical model allows to validate spring washer settings without expensive experimental tests and on the other hand to predict the damper force. In turn, a new hydraulic damper project is launched into the market

The aim of the paper is to show the impact of complexity of numerical model on the analysis results. System model of entire hydraulic damper created by the linear model, 2D nonlinear model-simplified models and 3D nonlinear model - advanced model.

2. Development of a valve and damper model

Valve system design and configuration determines the characteristic of damping force vs. rod velocity and damping force vs. rod displacement which are specified during the train bogie design process.

The hydraulic damper characteristics are result of the train bogie design process. On the other hand, the major valve system durability contributor is the highest thickness of a valve spring washer. A valve system requires an adjustment process to achieve the damping forces at specified velocities within the given tolerance band (typically 15%). The adjustment process is mostly manually conducted by a trained operator in the prototype workshop using customized spring washers of different diameters and thicknesses. The objective is to meet the customer damping force while minimize the stress level trying to reduce the spring washer thickness manipulating the number of spring washers and their diameters.
A typical damping force calibration process consists of the following steps:

- rebuilding the piston valve (change in number of spring washers, their diameter, or thickness);
- rebuilding the base valve (change in number of spring washers, their diameter, or thickness);
- changing the oil volume in the damper if an aeration effect occurred.

Damping force calibration can be carried out using a numerical method. Created three types of valve system models. These are simplified linear (Matlab), simplified nonlinear (Matlab) and advanced non-linear (Ansys Workbench) models. Model simplification refers to geometrical model representation. A simplified spring washer stack model assumes flexural rigidity assigned to annular finite elements. The most general discretization is provided through application of a grid of finite elements in an advanced model created utilizing a general-purpose software package Ansys Workbench. Results of simulations performed with the advanced model take into account influence of a valve assembly.

2.1. Linear model

Simplified linear model considers only the most essential components of a valve system, i.e. a hub (clamping edge) determining the clamping diameter as a rigid element, spring washers of different diameters and thicknesses, uniform pressure load or force and a land (valve seat) defining the area to which the pressure is applied (Fig. 1).

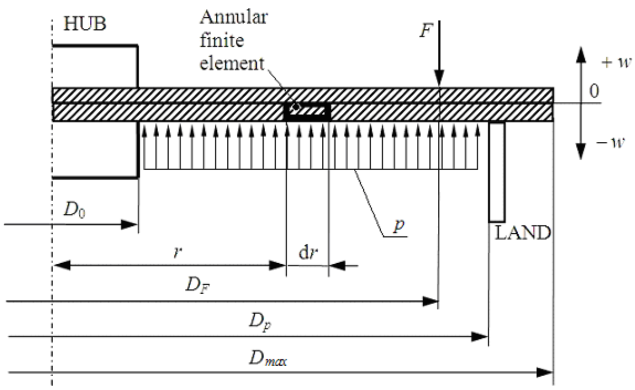

Fig. 1. Geometry and load distribution layout (D0-clamping diameter [m], DF-diameter of applied force [N], Dp-land (hydraulic) diameter [m], Dmax maximal diameter of spring washer [m], F-load force [N], p-load pressure [Pa], r-polar coordinate of finite element, w-vertical deflection [m]) [5]

The complete derivation of mathematical relations for the linear model has been presented in [5]. Finally, obtained are:
(i) the deflection w

$$w(r)=\frac{1}{9}\frac{F}{2\pi RD}r^3+\frac{1}{9}\frac{p}{2RD}R_p^2r^3-\frac{1}{75}\frac{p}{2RD}r^5+\ldots$$
$$\ldots+\frac{1}{4}C_1r^2+C_2\ln(r)+C_3. \qquad (1)$$

(ii) the slope dw/dr

$$\frac{dw(r)}{dr}=\frac{1}{3}\frac{F}{2\pi RD}r^2+\frac{1}{3}\frac{p}{2RD}R_p^2r^2-\frac{1}{15}\frac{p}{2RD}r^4+\frac{1}{2}C_1r+C_2 \qquad (2)$$

,

(iii) the radial moment M_r

$$M_r(r)=-D\left[\frac{1}{3}\frac{F}{2\pi RD}r(2+\nu)+\frac{1}{3}\frac{p}{2RD}R_p^2r(2+\nu)-\ldots\right.$$
$$\left.\ldots-\frac{1}{15}\frac{p}{2RD}r^3(4+\nu)+\frac{1}{2}C_1(1+\nu)+C_2\frac{1}{r^2}(-1+\nu)\right] \qquad (3)$$

Equations of the simplified model have been implemented in Matlab so that their solutions can be presented in a graphical and textural form.

2.2. Simplified nonlinear model

The nonlinear model is an extension of the linear one; for details concerning both models refer to [6]. The nonlinear model takes into account two variables, radial and perpendicular strains. Equations above are rewritten as a system of five, first-order differential equations [7]. At each transition between annular finite elements, the value of the actual curvature and the radial strain has to reflect the change in the thickness [6].

$$\frac{d^2w_{i+1}}{dr^2}=-\nu\frac{dw_i}{dr}+\frac{D_i}{D_{i+1}}\left(\frac{d^2w_i}{dr}+\nu\frac{dw_i}{dr}\right) \qquad (4)$$

$$\frac{du_{i+1}}{dr}=-\frac{1}{2}\left(\frac{dw_i}{dr}\right)^2-\frac{\nu\cdot u_i}{r}+\frac{t_i}{t_{i+1}}\left[\frac{du_i}{dr}+\frac{1}{2}\left(\frac{dw_i}{dr}\right)^2+\frac{\nu\cdot u_i}{r}\right] \qquad (5)$$

The first constraint, equation (4), results from the equality of the moment while the second, equation (5), results from the equality of the force. The displacement, the slope and the radial displacement are unchanged at each transition. This system of equations can be solved for the set of given initial conditions, i.e. displacement, slope, curvature, radial displacement and strain at the clamping radius. For a rigid clamping the displacement and the radial displacement are both equal to 0 by definition. The slope is known and defined by the geometrical relations in the piston. Two other initial conditions are unknown and have to be found iteratively using the linear model to improve the accuracy of the initial guess. Such an approach was first proposed in [6].

2.3. Advanced nonlinear model

Advanced nonlinear simulation was conducted with the use of finite element methods in ANSYS Workbench 12.1. The piston component was modeled as deformable part, while the spring washers as elastic part with properties listed in Table 1. The large-displacement solver was involved to increase nonlinear effects occur at high pressure load. The contacts among particular components were defined.

The sensitivity analysis was performed to determine the best mesh density. The Quadratic Tetrahedron (Mechanical APDL Name: Mesh200) finite elements were used in simulation.

The following boundary conditions were applied (Fig. 9):

- axisymmetrically fixation of the spring washers removing the rotational and vertical movement (Cylindrical Support);
- pre load force of the threaded nut 120N (on disk washer);
- equivalent oil pressure load of 5MPa, increasing linearly with the span of 0.5MPa;

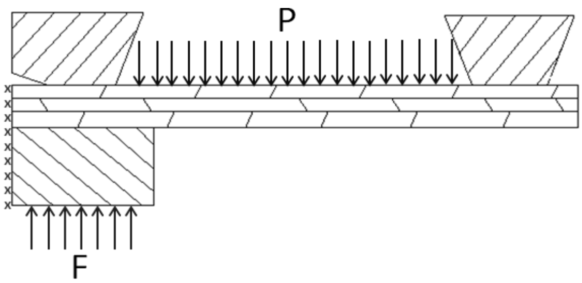

Fig. 2. The boundary conditions assumed in the FE model

There are two steps essential to loading and unloading a spring washer stack in the model:

- applying preload;
- applying the loading pressure.

During the preload step, the rod nut (rigid part) is moved down, while the piston hub (rigid) is held fixed. The nut moves until the clamping force is equal to the preload force resulted from the thread reaction (120N). In the second step, the oil pressure equivalent load is applied to the spring washers stack during the rebound cycle.

3. Simplified models vs. advanced valve system model

The models described in the previous subsections were used to determine the detailed model of the valve (hydraulic characteristics: pressure-volume flow and the characteristics of strength: stress-the pressure/force damping).

The spring washer stack configuration is presented in Table 1 and Table 2, respectively for piston and base vale. The spring washer mechanical properties are presented in Table 3.

Table 1.
Piston valve spring washer stack configuration

Item	Component name	Spring washer dimensions
1	elastic spring washer 0.2	Ø32 x Ø16 x 0.2
2	elastic spring washer 0.3	Ø32 x Ø16 x 0.3
3	elastic spring washer 0.3	Ø32 x Ø16 x 0.3
4	disk washer	Ø20 x Ø16 x 2 | r = 0.7

Table 2.
Base valve spring washer stack configuration

Item	Component name	spring washer dimensions
1	elastic spring washer 0.2	Ø 22 x Ø 6.2 x 0.2
2	elastic spring washer 0.2	Ø 22 x Ø 6.2 x 0.2
3	disk washer	Ø 16 x Ø 6.2 x 2 | r = 0.7

Table 4 shows a comparison of properties of the simplified valve model and advanced valve model. A significant differences can be seen in the simulation time. Simulation time for advanced models is much longer than for simplified models.

Table 3.
Mechanical properties of spring washer used in valve systems

Parameter	Value	Unit
Young's modulus	210000	MPa
Poisson ratio	0.3	[-]
Yield strength ($Re_{min.}$)	250	MPa
Tensile strength Rm	600-950	MPa
Hardness max.	215	HB

Table 4.
Comparison of properties of valve system models

Model properties	Linear simplified model	Nonlinear simplified model	Advanced model
Spring washer clamping	rigid	rigid	elastic
Land material	rigid	rigid	elastic
External friction	no	no	yes
Plasticity	no	no	yes
Spring washer material parameters	same for each spring washer	same for each spring washer	may be different for each spring washer
Simulation time	<5 sec	<60 sec	<8h
Large deformation	no	no	yes

3.1. Detailed model of the valve

The detail numerical valve model allows to obtain the hydraulic characteristic (differential pressure across the valve vs. volumetric oil flow) and durability characteristic (averaged/max stress level in a stack of spring washers vs. differential pressure across the valve).

Table 5.
The critical values of selected parameters of the analyzed valves for simplified and advanced model

Model	Displecement [mm]	Stress [MPa]
The piston valve		
simplified linear	0.719	1626.7
simplified nonlinear	0.339	1191.5
advanced	0.406	1363.7
The foot valve		
simplified linear	0.262	1403.0
simplified nonlinear	0.114	915.2
advanced	0.118	985.5

The hydraulic characteristic is used in a system model while the durability characteristic allows to verify the critical stress level for the given load of a hydraulic damper.

The results are presented in Fig. 3 and 4 in a form of displacement vs. the applied equivalent oil pressure load. The displacement is obtained in the cross section of a spring washer above the supporting piston edge.

Table 5 presents the critical values of displacement and stress of the analyzed valves for simplified linear/nonlinear and advanced model.

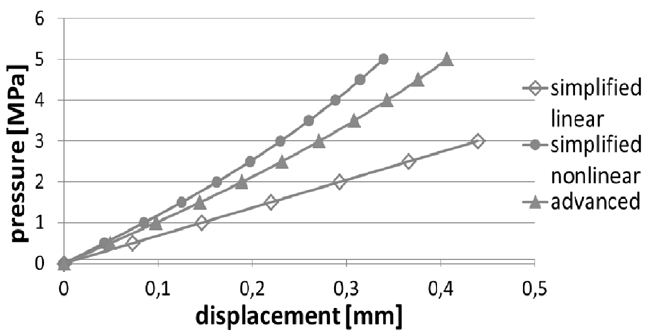

Fig. 3. Comparison of the simplified linear/nonlinear model and advanced model for piston valve

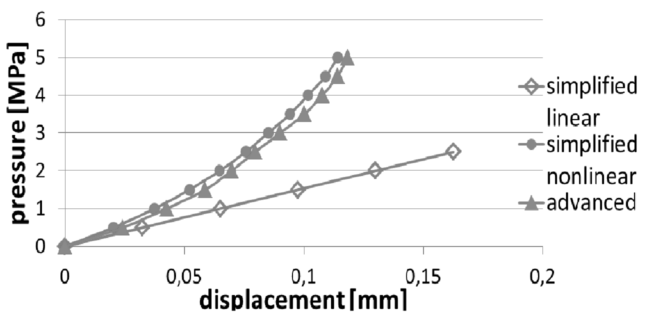

Fig. 4. Comparison of the simplified linear/nonlinear model and advanced model for foot valve

3.2. System model of the hydraulic damper

A significant improvement in a hydraulic damper validation process involves a model-based approach which allows to obtain the most durable spring washer settings regarding the minimum stress optimum criterion. A model-based approach requires to formulate two models, namely the system model of entire hydraulic damper and detail numerical 2/3 D model of the spring washers stack including the boundary conditions (subsection 3.1). The work [8] presents a model-based approach to understand the hydraulic damper operation at the electrical locomotive.

The system model provides damping forces using the following formula [9]:

$$F_d = p_{reb} \cdot A_{reb} + p_0 \cdot A_{rod} - p_{com} \cdot A_{com} \qquad (6)$$

where:
F_d-damping force generated by the hydraulic damper;
A_{rod}, A_{com}, A_{reb}-surfaces of the piston (rod, rebound, compression), [m^2];
p_{com}, p_{reb} -t he pressure in the compression and rebound chambers, [Pa];
p_0-atmospheric pressure, $p_0 = 1e^5$ [Pa].

The valve opening vs. equivalent oil pressure load characteristics for simplified and advanced model were obtained using approximation formulas described equations (Table 6).

Table 6.
Approximations equation for characteristics pressure-displacement for the models tested

Simplified linear model	
The piston valve	y = 6.8311x - 3E-14
The foot valve	y = 15.382x - 7E-15
Simplified nonlinear model	
The piston valve	$y = 14.068x^2 + 9.7608x + 0.0369$
The foot valve	$y = 246.12x^2 + 14.058x + 0.0643$
Advanced nonlinear model	
The piston valve	$y = 8.1372x^2 + 8.8257x + 0.0311$
The foot valve	$y = 258.35x^2 + 9.7193x + 0.0574$

The hydraulic characteristic (differential pressure across the valve vs. volumetric oil flow) were obtained in the second step as presented in Fig. 16. The volumetric flow rate was determined using the following formula:

$$q = C_d \cdot (\pi \cdot d \cdot x) \cdot \left(\frac{2p}{p} \right)^{\frac{1}{2}} \qquad (6)$$

where:
q – flow rate through valve [m^3/s];
p – pressure drop across valve assembly [Pa];
Cd – flow (discharge) coefficient for valve = 0.35;
ρ - fluid density = 850 [kg/m^3];
x – valve disk lift [m];
d – the outflow valve diameter = 0.029 [m] (piston valve) and 0.020 [m] (foot valve).

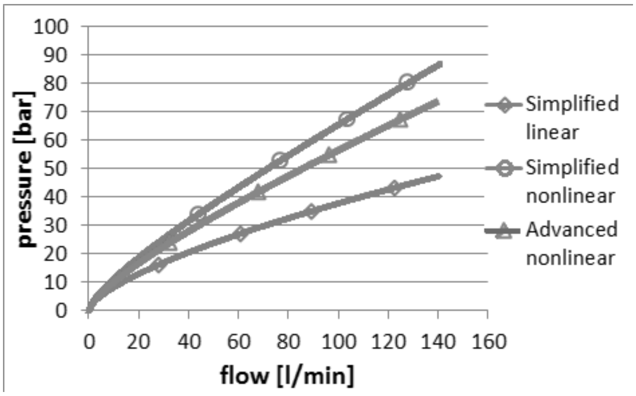

Fig. 5. The hydraulic characteristics for piston valve for the models tested

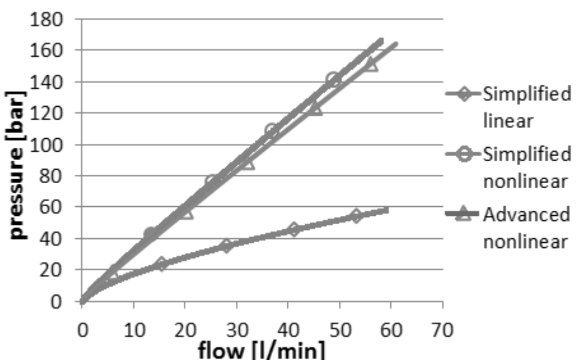

Fig. 6. The hydraulic characteristics for foot valve for the models tested

The obtained valve characteristics allowed to compute damping forces based on the formula (6) representing the damper system model, respectively for rebound and compression stroke. Model parameters were listed in Table 7.

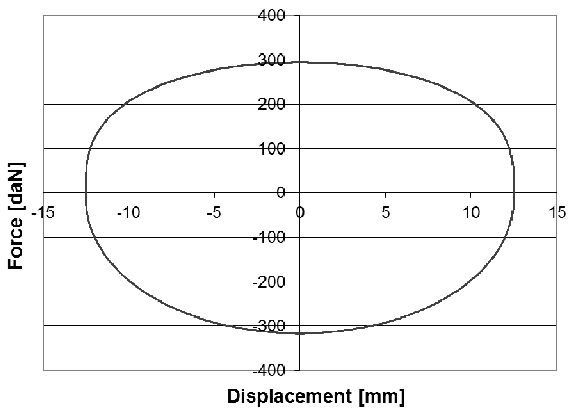

Fig. 7. Damping forces graph for the entire system model of [4]

Table 7.
The parameters used to determine the damping forces of the hydraulic damper

Parameter	Value	Unit
Rebound area	0.001433	m^2
Compression area	0.001963	m^2
Rod area	0.000531	m^2
Stroke	25.0	mm
Velocity	0.20	m/s

The damping force for advance model is presented for the selected velocity v=0.2m/s as a diagram force vs. piston rod displacement (Fig. 7).

The results obtained with the numerical model were compared with the experimental results obtained with the use of servo-hydraulic MSP25 IST tester. The relative error between simulated and measured force-displacement curve was calculated as [9]:

$$E_r = \frac{\sum F_d - \sum F_c}{\sum F_d} \cdot 100\% \quad (7)$$

where:

F_d - the expected value of the force (with experimental measurements) [daN];

F_c - the calculated value of force (measured number) [daN];

E_r - relative error [%].

The flow coefficient in formula (6) was additionally adjusted to minimize the relative error.

Table 8.
Comparison of tested models with the experimental analysis. A-Simplified linear model; B-Simplified nonlinear model; C-Advanced nonlinear model; D-experimental model

Damping force	A	B	C	D
Unit	[daN]	[daN]	[daN]	[daN]
Rebound	241	329	295	300
Compression	152	338	317	320

Table 9.
Errors for the analyzed results compared to the experimental model

Error	A	B	C
Unit	[%]	[%]	[%]
Rebound	19.6	9.6	1.6
Compression	52.5	5.6	0.9

4. Fatigue model

Theoretical issues fatigue model for hydraulic damper are presented in [10]. A Wöhler diagram allows to visualize the number of cycles to failure N versus the load amplitude S and the material parameters α and β estimated from the Basquin's relation [11]:

$$N = \alpha \cdot S^{-\beta} \tag{8}$$

The resulting estimates α^ and b^ are regarded as a characterization of the fatigue property of the material at the specified R-ratio.

The Palmgren-Miner (PM) linear damage hypothesis is most often applied to obtain the resultant damage [11] (graphically shown in Fig. 8). This is described in detail in [10, 11].

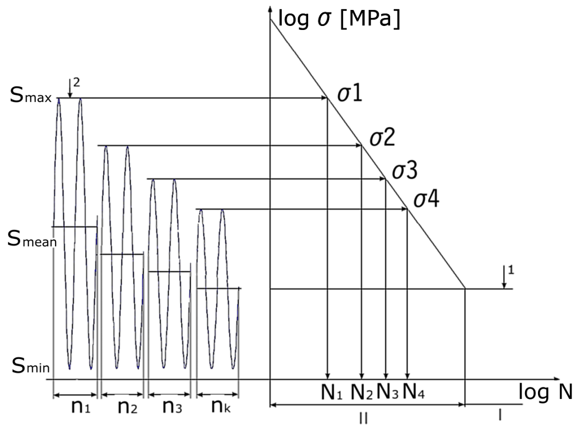

Fig. 8. Graphical interpretation of Palmgren-Miner linear damage hypothesis [10]

The identification of the fatigue model is performed using a constant or variable amplitude test data using the strain/stress model. The predicted stress α_{sim} and the measured force F_{mes} are interpolated and mapped using the following function:

$$\sigma_{sim} = f(F = F_{mes}) \tag{9}$$

This requires to consider the force applied to the tested spring washer stack in the stress-strain model. The calibrated stress-strain model is used to predict the critical number of cycles before the spring washer stack failure as follows:

$$n = f(\sigma_{sim}) \tag{10}$$

Equation 10 has visual representation in the form of a Wöhler diagram. Wöhler diagram for elastic spring washer with a thickness of 0.3 mm are presented in Fig. [10]. The graph was generated from experimental measurements [12].

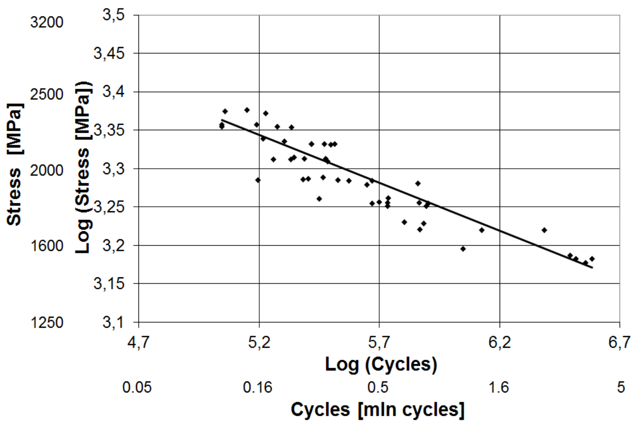

Fig. 9. Experimental Wöhler curves [12]

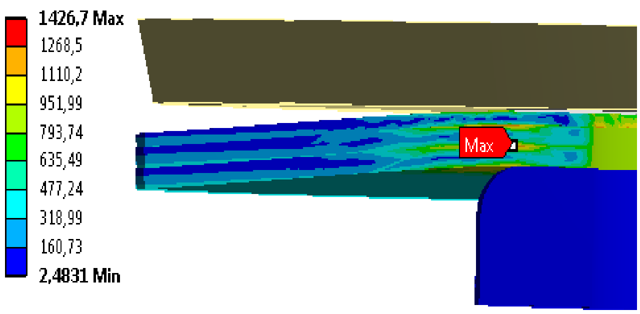

Fig. 10. Maximum stress in stack of disks of piston valve

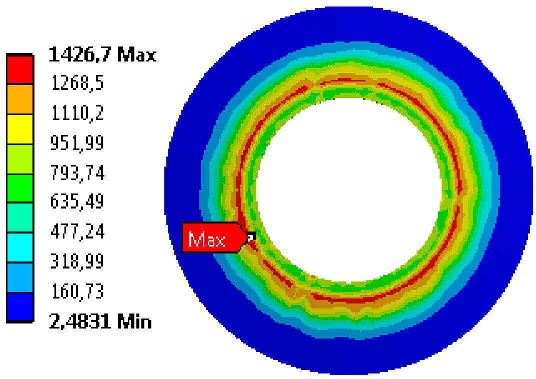

Fig. 11. Maximum stress in elastic spring washer 0.3mm

The unlimited fatigue endurance was recorded for stress value about 1300MPa. The stress value for the analyzed detailed models of the valve are shown in Table 5.

Figure 10 shows the map of the stress for advanced model developed with the use of FEM analysis in Ansys Workbench program. It was observed that the highest stress (the probability of failure) It is in an area away from the center of the spring washer by an amount equal to the radius of the disk washer (Fig. 11).

5. Summary and conclusions

The paper presents a model-based method to configure and verify railway hydraulic damper equipped with spring washer based valve systems. Three simulation models are discussed, namely: a linear model, a simplified nonlinear model, and an advanced nonlinear model. The method consists of the following steps: (i) model configuration for a specific valve system, (ii) stress/opening/flow vs. pressure calculation and (iii) fatigue vs. stress calculation.

Simplified linear and simplified nonlinear models are stated in an explicit form as a set of equations derived from the theory of bending circular plates for small and large plate deflections, respectively [7]. A number of assumptions underlying this approach justifies calling the models "simplified". Simplified models may be applied if the results have to be obtained within a limited time frame and the results are slightly less accurate compare to the advance model.

Simplified models were developed using Matlab software. The advanced nonlinear model was developed using the finite element method (FEM) in ANSYS Workbench software. The model includes deformation of the all components (like spring washer clamping, valve seat material and spring washer) and mechanical properties of used valve spring washer may be different for each spring washer (Table 4). Friction forces acting between the sliding spring washers are also considered in this model.

The simulation times are collected in Table 4. The advanced nonlinear model requires a much longer simulation time compared to the linear simplified one considering the simulation case of the spring washers of the piston valve under 5MPa load pressure (for linear simplified model 5 seconds and for advanced model 8 hours). Advanced nonlinear model is the most accurate, since rebound and compression forces calculated by this model differ only about 1.6% and 0.9% from the values obtained experimentally.

The final step of the model-based hydraulic damper verification is to determine the number of cycles of life for spring washers. The number of cycles was calculated based on experimentally obtained Wöhler curve. A spring washer has the unlimited service life if stress value is not greater than 1300MPa.

Linear model overestimate the life time until spring washer failure since it predicts 2 mln cycles., while nonlinear models indicate the unlimited service life (Tabele 5). The analysis was performed for the velocity v = 0.2m/s. For higher velocity values (increase of 1m/s) of damping force will be higher, making the value of stress in the disk will also increase by about 300MPa, as for the analyzed case resulting in fewer cycles (limited service life equal 2 mln. of cycles).

It follows that the developed advanced model enables to determine critical velocity, at which can be obtained unlimited durability for spring washers. FEM analysis also shows the location where the damage occurs at a spring washers. The damage (i.e. large deformation, crack) occurs at a radius equal to the radius of the spring washer (Fig. 11).

For foot valve the stress value in disks is much lower (985.5MPa for advanced model), so the number of life cycles is much higher (unlimited).

The reason is the smaller spring washers diameter and the larger diameter of the spring washer clamping for foot valve as compared to the valve piston spring washers.

The proposed three step hydraulic damper verification method was numerically validated. The analysis is more accurate using an advanced nonlinear model.

Nomenclature

D – flexural rigidity of a spring washer [$Pa \cdot m^3$]
D_0 – clamping diameter [m]
D_F– diameter of applied force [m]
D_{max} – maximal diameter of spring washer stack [m]
D_p – land (hydraulic) diameter [m]
dr– length of finite element [m]
E– Young modulus [MPa]
E_{rr} – relative error [%]
F – load force [N]
F_c – calculated force value [N]
F_d – desired force value (measured) [N]
h – spring washer thickness [m]
M_0– bending moment per unit length [N·m/m]
M_r – radial bending moment per unit length [N·m/m]
M_t – tangential moment (shear) per unit length [N·m/m]
N_r – radial tensile force per unit length [N/m]
N_t – tangential tensile force per unit length [N/m]
p – load pressure [Pa]

q– flow [m^3/s]
Q– applied shear force per unit length [N/m]
Q_F – applied shear force corresponding to load force per unit length [N/m]
Q_p – applied shear force corresponding to load pressure per unit length [N/m]
Q_r – shear force per unit length [N/m]
q_r – lateral load [N/m]
r – polar coordinate of finite element
R – external radius of finite element (constant) [m]
r_n – projection of polar coordinate on the normal direction to the finite element [m]
R_p – external radius of applied pressure [m]
r_t – projection of polar coordinate on the finite element plane [m]
u – radial displacement of axisymmetrical finite element [m]
v – velocity [m/s]
w – vertical deflection [m]
X – spring washer deflection at land diameter [m]
Δp – pressure drop across the valve assembly [Pa]
ε_r – strain in the radial direction [m]
ε_t – strain in the tangential direction [m]
Θ – polar coordinate of finite element
ν – Poisson ratio (ν = 0.3)

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