POLISH ACADEMY OF SCIENCES - MATERIALS SCIENCE COMMITTEE
SILESIAN UNIVERSITY OF TECHNOLOGY OF GLIWICE
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Conference Proceedings

11th INTERNATIONAL SCIENTIFIC CONFERENCE
ACHIEVEMENTS IN MECHANICAL & MATERIALS ENGINEERING

Continued fraction expansion method represented by polar graphs as the tool of modification of discrete and continuous mechanical systems*

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In this paper the method was applied in order to modiffication of the dynamical characteristics of the longitudinally or torsionally transforming vibration subsystems with cascade structure called the continued fraction expansion method. Next a way of determination of the formula for the dynamical flexibility of the transforming vibration subsystems with a cascade structure has been presented. Obtained forms for the dynamical characteristics determinate a base to theirs programming in order to obtain the dynamical flexibilities for n-bars longitudinally or torsionally transforming vibration subsystems.

1. INTRODUCTION

Using the symbols used in [3-6, 9], a following couple

$$X=\left({}_1X,\ {}_2X\right) \tag{1}$$

is called a *graph,* where: ${}_1X=\{{}_1x_0,\ {}_1x_1,\ {}_1x_2,\ \ldots,{}_1x_n\}$ -finite set of vertices, ${}_2X=\{{}_2x_1,\ {}_2x_2,\ \ldots,\ {}_2x_m\}$-family of edges, being two-element subsets of vertices, in the form of ${}_2x_k=\left({}_1x_i,\ {}_1x_j\right)$ $(i,j = 0,1, \ldots ,n)$ (of. [2]).

The couple

$${}^kX=\left({}_1X,\ {}_2^kX\right) \tag{2}$$

is called a *hypergraph,* where: ${}_1X$ is the set as in (1), and ${}_2^kX=\left({}_2^kX^{(i)}/i\in N\right)$, ($k$=2,3, ... $\in N$) is a family of subsets of set ${}_1X$; the family ${}_2^kX$ is called a *hypergraph* over ${}_1X$ as well, and ${}_2^kX=\{{}_2^kX^{(1)},\ {}_2^kX^{(2)},\ \ldots,{}_2^kX^{(m)}\}$ is a set of edges [2], called *hyperedges* or *blocks,* if

* This work has been conducted as a part of the research project No. 5 TO7C 029 23 supported by the Committee of Scientific Research in 2002 2004.

$${}^{k}_{2}X \neq \varnothing \; (i \in N), \tag{3}$$

$$\bigcup_{i \in I} {}^{k}_{2}X^{(i)} = {}_{1}X, \; (I \subset N). \tag{4}$$

Using notion of graph and of hypergraph and their connections with structural numbers [1-6,9], methods of modification of transforming vibration system as task of the synthesis of dynamical characteristic - mobility has been presented.

A characteristic - dynamical flexibility is given in form

$$Y(s) = \left(\sum_{i=0}^{k} c_i th^j \Gamma s \right) : \left(s \sum_{j=0}^{l} d_j th^j \Gamma s \right). \tag{5}$$

After Wyndrum's and Richards' transformations [7,8] the mobility has been obtained as

$$V(s) = \sum_{i=0}^{k} c_i r^i : \sum_{j=0}^{l} d_j r^j . \tag{6}$$

where: $c_k, c_{k-1}, \ldots, c_0, d_l, d_{l-1}, \ldots, d_0$ are any real numbers, $\Gamma = \sqrt{\frac{\rho}{G}} L = \sqrt{\frac{\rho^{(i)}}{G^{(i)}}} L^{(i)}$, ρ -mass density,

G-Kirchhoff's modulus, $L = L^{(i)}$ - length of basic element, $s = j\omega$, $j = \sqrt{-1}$, $c_k, c_{k-1}, \ldots, c_0, d_l, d_{l-1}, \ldots, d_0$ - real numbers, i,j,k,l - natural numbers, k

2. CONTINUED FRACTION EXPANSION METHOD OF THE REVERSE TASK OF THE TRANSFORMING VIBRATION SUBSYSTEMS REPRESENTED BY GRAPHS

The method of the synthesis of transformed mobility function $V(r)$ is presented here, assuming the even number of elements, and when k is an even natural number, then $V(r)$ takes form

$$V(r) = \frac{c_k r^k + c_{k\text{-}1} r^{k-1} + \ldots + c_0}{d_{k\text{-}1} r^{k-1} + d_{k\text{-}3} r^{k-k} + \ldots + d_1 r} \tag{7}$$

After dividing in (7) the numerator by denominator - it is a first step of the synthesis - the equation below is obtained

$$V(r) = V_r^{(1)}(r) + \frac{L_{k-2}(r)}{M_{k-1}(r)} = V_r^{(1)}(r) + \frac{1}{\frac{M_{k-1}(r)}{L_{k-2}(r)}} = V_r^{(1)}(r) + \frac{1}{U_2(r)} = \frac{r}{c_r^{(1)}} + \frac{1}{U_2(r)}, \tag{8}$$

where: $c_r^{(1)}$ is value of "i" synthesized discrete elastic element.

The second step is the realization of the function $U_2(r)$ into (8). When dividing $M_{k-1}(r)$ by $L_{k-2}(r)$, $U_2(r)$ takes form

$$U_2(r)=U_z^{(2)}(r)+\frac{M_{k-3}(r)}{L_{k-2}(r)}=U_z^{(2)}(r)+\frac{1}{\dfrac{L_{k-2}(r)}{M_{k-3}(r)}}=U_z^{(2)}(r)+\frac{1}{V_3(r)}=J_z^{(2)}r+\frac{1}{V_3(r)}\ , \tag{9}$$

where: $J_z^{(i)}$ -value of "i" synthesized discrete inertial element.

The mobility function after operations (8-9) is given in the following form

$$V(r)=V_r^{(1)}+\frac{1}{U_z^{(2)}(r)+\dfrac{1}{V_3(r)}}\ . \tag{10}$$

The third step is the realization of the mobility function $V_3(r)$ into (10) as

$$V_3(r)=V_r^{(3)}(r)+\frac{L_{k-4}(r)}{M_{k-3}(r)}=V_r^{(3)}(r)+\frac{1}{\dfrac{M_{k-3}(r)}{L_{k-4}(r)}}=V_r^{(3)}(r)+\frac{1}{U_4(r)}=\frac{1}{c_r^{(3)}}+\frac{1}{U_4(r)}\ . \tag{11}$$

Finally the mobility (7) as a continued fraction is obtained in form

$$V(r)=V_r^{(1)}+\cfrac{1}{U_z^{(2)}(r)+\cfrac{1}{V_r^{(3)}(r)+\cfrac{1}{U_z^{(4)}(r)+\cdots+\cfrac{1}{V_r^{(k-1)}(r)+\cfrac{1}{U_z^{(k)}(r)}}}}}=\frac{r}{c_r^{(1)}}+\cfrac{1}{J_z^{(2)}r+\cfrac{1}{\cfrac{r}{c_r^{(3)}}+\cfrac{1}{J_z^{(4)}r+\cdots+\cfrac{1}{\cfrac{r}{c_r^{(k-1)}}+\cfrac{1}{J_z^{(k)}}}}}}\ . \tag{12}$$

The above formulas and sentences in [3] are a base for the computer aided synthesis of longitudinally or torsionally transforming vibration subsystems by the continued fraction expansion method.

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