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Theoretical-analytic development of a representative multi-body system of the static behavior of the human body

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1 INTRODUCTION

The phenomenous that they manifest risk of generating physical problems in sedence are related with all those physical magnitudes that somehow characterize the state of thermodynamic and mechanic balance of a person settles down with her contact environment. In reference it can understand each other without more difficulty that those physical magnitudes related with the process of thermodynamic balance will act a priori long term to be this a slow process (temperature, the humidity and the PH), on the contrary, the processes of damping mechanical balance like ours produces balanced times much smaller, being given by it in an immediate way, the tensional state associated to the balance.

The problem that thinks about is the one of determining the articular reaction values and superficial load. In the following pages it will be exposed the parametrization of the dynamic-dimensional properties of a person and the opportune modelation of their articular bonds, to pay attention later in the position of the mechanical problem of determination of reactions, obtaining a representative analytic model of the human body in static situations.

With this parametrization we can concentrate all the necessary information on this case only in two values that will be the Antropometric Percentile and the IMC. On this way it would subtract to specify the adopted corporal position. It should be clear that the first objective of this study is to determine the magnitudes of the components of the forces and torques located in each articulation in function of the person and the posture adopted.

2 ANALYSIS AND DYNAMIC-DIMENSIONAL DEFINITION

The outlined multibody human system decreases to the existence of 18 members, being only 11 useful in this problem for symmetry reasons. In the next pages it will be obviated how they depends on:

- Longitude of each member
- Mass of each member
- Position of the center of masses of each member

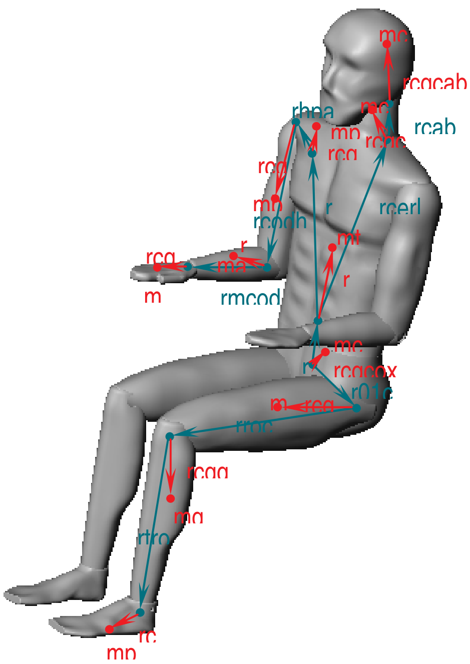
Of each individual's characteristic parameters to study:

- Antropometric Percentile
- Index of corporal mass IMC

Where:

$[r_{roc}]$ = vector of position relative knee-hip referred to the axes of the thigh in the hip
$[r_{tro}]$ = vector of position relative ankle-knee referred to the axes of the leg in the knee
$[r_{cerl}]$ = vector of cervical-lumbar relative position referred to the axes of the back in the lumbar one
$[r_{cabcer}]$ = vector of relative position atlas-cervical1 referred to the axes of the neck in the cervical1
$[r_{pall}]$ = vector of palette-lumbar relative position referred to the axes of the back in the lumbar one
$[r_{hpal}]$ = vector of position relative shoulder-palette referred to the palette's axes in the union
$[r_{codh}]$ = vector of position relative elbow-shoulder referred to the axes of the arm in the shoulder
$[r_{mcod}]$ = vector of position relative doll-elbow referred to the axes of the forearm in the elbow
$[r_{o1c}]$ = vector of position relative hip-origin referred to the absolute axes
$[r_{lc}]$ = vector of position relative lumbar-hip referred to the absolute axes
$[r_{cgmus}]$ = vector of position of the center of masses of the thigh referred to their axes in the hip
$[r_{cgg}]$ = vector of position of the center of masses of the leg referred to their axes in the knee
$[r_{cgp}]$ = vector of position of the center of masses of the foot referred to their axes in the ankle
$[r_{cgt}]$ = vector of position of the center of masses of the trunk referred to their axes in the lumbar one
$[r_{cgc}]$ = vector of position of the center of masses of the neck referred to their axes in the cervical1
$[r_{cgcab}]$ = vector of position of the center of masses of the head referred to their axes in the atlas
$[r_{cgpal}]$ = vector of position of the center of masses of the palette referred to their axes in the union
$[r_{cgb}]$ = vector of position of the center of masses of the arm referred to their axes in the shoulder
$[r_{cga}]$ = vector of position of the center of masses of the forearm referred to their axes in the elbow
$[r_{cgcox}]$ = vector of position of the center of masses of the coxis referred to the absolute axes
$[r_{cgman}]$ = vector of position of the center of masses of the hand referred to their axes in the doll
m_m = mass of the thigh
m_g = mass of the leg
m_p = mass of the foot
m_t = mass of the trunk
m_c = mass of the neck
m_{cab} = mass of the head
m_{pal} = mass of the palette
m_b = mass of the arm

m_a = mass of the forearm
m_{man} = mass of the hand
m_{cox} = mass of the coccix
e_c = thickness of the head
e_e = thickness of the back
e_a = thickness of the forearm
e_n = thickness of the hand
e_m = thickness of the thigh
e_g = thickness of the leg
e_p = thickness of the foot



It will be omitted the parametrization of each one of the vectors and previous magnitudes in function of the sex, antropometric percentile and IMC.

3 ANALYSIS AND ARTICULAR DEFINITION

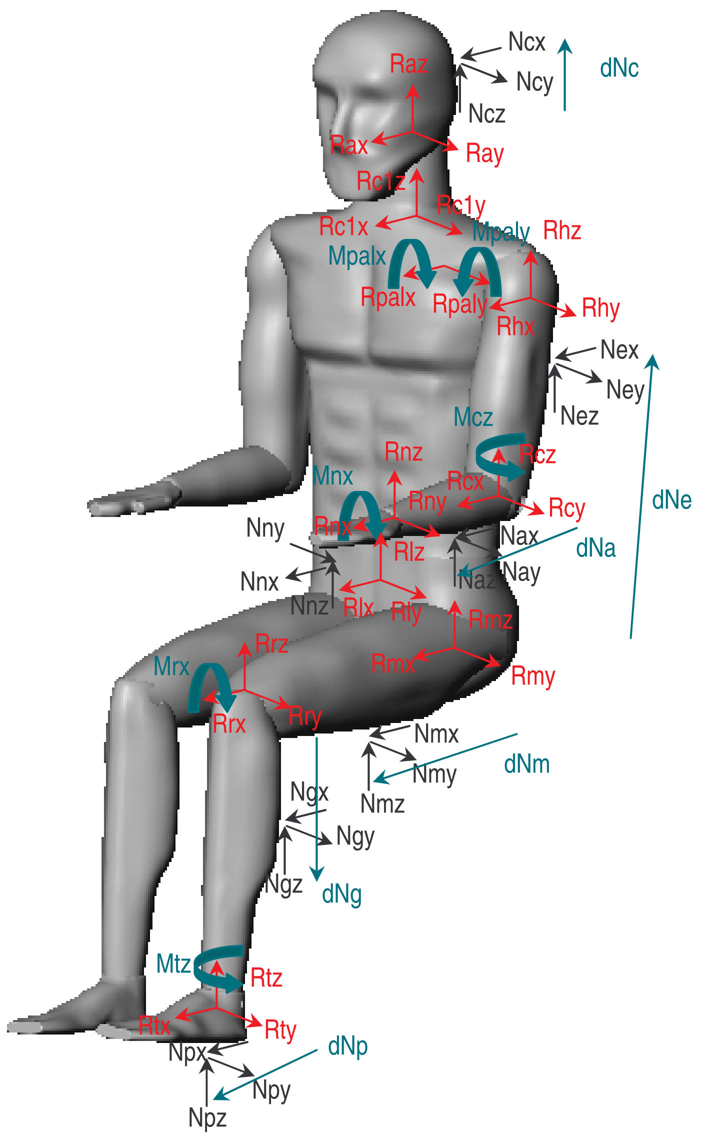
In the case of the articular relationships the bonds settle down for contact of complex geometric configurations, the existence of ligaments... however there are very approximate simple modelations of each one, that will be able to help to simplify the position of the mechanical problem of total balance. Finally there are only 14 angular parameters of corporal positioning that are being of importance in the sedence position:

Thigh = α_m, β_m, ϕ_m Atlas = αa
Back = αe Knee = αr
Ankle = αt Cervical1 = αcer

Shoulder = αh, βh, ϕh Elbow = αc
Doll = αn Palette = ϕp

4 EXHIBITION OF THE POSITION OF THE PROBLEM

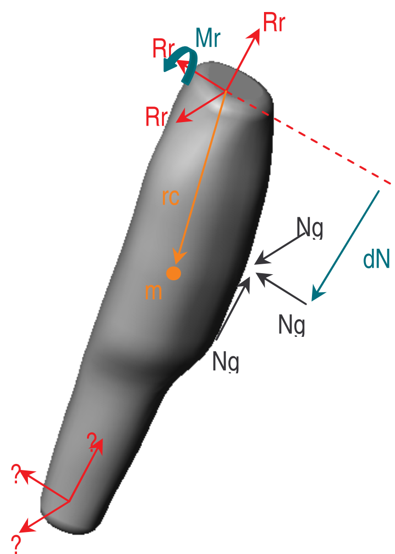
It is necessary to outline the mechanical problem of balance for the mentioned multibody system. So we will analyze their dynamic state based on the application of the Newtonian mechanics, looking for the articular and external reactions that balance in a global way the entirety of the group. This reactions are collected in the following graph:



Finally there are 66 unknowns, coinciding without great value that the number of equations is 66 too, for counting 11 members and six balance equations for each one of them, of forces and torques in the three axes. The 66 unknowns are the following ones:

Ncx, Ncy, Ncz = Components of the reaction in the support of the head
dNc = Position of the reaction of the support of the head
Nax, Nay, Naz = Components of the reaction in the support of the forearm
dNa = Position of the reaction of the support of the forearm
Nnx, Nny, Nnz = Components of the reaction in the support of the hand
dNn = Position of the reaction of the support of the hand
Nmx, Nmy, Nmz = Components of the reaction in the support of the thigh
dNm = Position of the reaction of the support of the thigh
Ngx, Ngy, Ngz = Components of the reaction in the support of the leg
dNg = Position of the reaction of the support of the leg
Npx, Npy, Npz = Components of the reaction in the support of the foot
dNp = Position of the reaction of the support of the foot
Nex, Ney, Nez = Components of the reaction in the support of the back
dNe = Position of the reaction of the support of the back
Rmx, Rmy, Rmz = Components of the internal reaction of the hip
Rrx, Rry, Rrz, Mrx = Components of the internal reaction of the knee
Rtx,Rty, Rtz, Mtz = Components of the internal reaction of the ankle
Rc1x, Rc1y, Rc1z = Components of the internal reaction of the articulation cervical1
Rax, Ray, Raz = Components of the internal reaction of the articulation of the atlas
Rpx, Rpy, Mpx, Mpy = Components of the internal reaction of scapular transition
Rhx, Rhy, Rhz = Components of the internal reaction of the shoulder
Rcx, Rcy, Rcz, Mcx = Components of the internal reaction of the elbow
Rnx, Rny, Rnz, Mnx = Components of the doll's internal reaction
Rlx, Rly, Rlz = Components of the internal reaction of the lumbar transition

Leg:



In the same way all the members are analyzed. We have outlined in vectorial form the 66 equations that define the problem. The system of equations is not lineal. This fact is easily observable if it is expressed in a general way , operating the vectorial form. For this we will take the following notation of implied matrix as well as the established notation for the independent terms:

$C(\alpha_a) = A^1$ $C(\alpha_t) = A^2$ $C(\alpha_m)\,C(\beta_m)\,C(\phi_m) = A^4$ $C(\alpha_n) = A^5$ $C(\alpha_c) = A^6$ $C(\alpha_h)\,C(\beta_h)\,C(\phi_h) = A^7$
$C(\phi_p) = A^8$ $C(-\phi_p) = A^9$ $C(\alpha_{cer}) = A^{10}$

The operated equations turn out to take the form:

$$Rax + Ncx = -b_1 \quad (1)$$

$$Ray + Ncy = -b_2 \quad (2)$$

$$Raz + Ncz = -b_3 \quad (3)$$

$$Ncy \cdot dNc = -b_4 \quad (4)$$

$$-Ncz \cdot e_c - Ncx \cdot dNc = -b_5 \quad (5)$$

$$-Ncy \cdot e_c = -b_6 \quad (6)$$

$$Rc1x - A^1_{11} . Rax - A^1_{12} . Ray - A^1_{13} . Raz = -b_7 \quad (7)$$

$$Rc1y - A^1_{21} . Rax - A^1_{22} . Ray - A^1_{23} . Raz = -b_8 \quad (8)$$

$$Rc1z - A^1_{31} . Rax - A^1_{32} . Ray - A^1_{33} . Raz = -b_9 \quad (9)$$

$$(- A^1_{21} . Rax - A^1_{22} . Ray - A^1_{23} . Raz)\, Z_{cabcer} - (- A^1_{31} . Rax - A^1_{32} . Ray - A^1_{33} . Raz)\, Y_{cabcer} = -b_{10} \quad (10)$$

$$(- A^1_{31} . Rax - A^1_{32} . Ray - A^1_{33} . Raz)\, X_{cabcer} - (- A^1_{11} . Rax - A^1_{12} . Ray - A^1_{13} . Raz)\, Z_{cabcer} = -b_{11} \quad (11)$$

$$(- A^1_{11} . Rax - A^1_{12} . Ray - A^1_{13} . Raz)\, Y_{cabcer} - (- A^1_{21} . Rax - A^1_{22} . Ray - A^1_{23} . Raz)\, X_{cabcer} = -b_{12} \quad (12)$$

$$Rtx + Npx = -b_{13} \quad (13)$$

$$Rty + Npy = -b_{14} \quad (14)$$

$$Rtz + Npz = -b_{15} \quad (15)$$

$$-Npy \cdot e_p = -b_{16} \quad (16)$$

$$Npz \cdot dNp + Npx \cdot e_p = -b_{17} \quad (17)$$

$$-Npy \cdot dNp + Mtz = -b_{18} \quad (18)$$

$$Rrx - A^2_{11} . Rtx - A^2_{12} . Rty - A^2_{13} . Rtz + Ngx = -b_{19} \quad (19)$$

$$Rry - A^2_{21} . Rtx - A^2_{22} . Rty - A^2_{23} . Rtz + Ngy = -b_{20} \tag{20}$$

$$Rrz - A^2_{31} . Rtx - A^2_{32} . Rty - A^2_{33} . Rtz + Ngz = -b_{21} \tag{21}$$

$$(- A^2_{21} . Rtx - A^2_{22} . Rty - A^2_{23} . Rtz) Z_{tr} - (- A^2_{31} . Rtx - A^2_{32} . Rty - A^2_{33} . Rtz) Y_{tr} - A^2_{13} \cdot Mtz + Mrz - Ngy \cdot dNg = -b_{22} \tag{22}$$

$$(- A^2_{31} . Rtx - A^2_{32} . Rty - A^2_{33} . Rtz) X_{tr} - (- A^2_{11} . Rtx - A^2_{12} . Rty - A^2_{13} . Rtz) Z_{tr} - A^1_{23} \cdot Mtz - Ngy \cdot e_g + Ngx \cdot dNg = -b_{23} \tag{23}$$

$$(- A^2_{11} . Rtx - A^2_{12} . Rty - A^2_{13} . Rtz) Y_{tr} - (- A^2_{21} . Rtx - A^2_{22} . Rty - A^2_{23} . Rtz) X_{tr} - A^2_{33} \cdot Mtz + Ngy \cdot e_g = -b_{24} \tag{24}$$

$$Rmx - A^3_{11} . Rrx - A^3_{12} . Rry - A^3_{13} . Rrz + Nmx = -b_{25} \tag{25}$$

$$Rmy - A^3_{21} . Rrx - A^3_{22} . Rry - A^2_{23} . Rrz + Nmy = -b_{26} \tag{26}$$

$$Rmz - A^3_{31} . Rrx - A^3_{32} . Rry - A^3_{33} . Rrz + Nmz = -b_{27} \tag{27}$$

$$(- A^3_{21} . Rrx - A^3_{22} . Rry - A^3_{23} . Rrz) Z_{roc} - (- A^3_{31} . Rrx - A^3_{32} . Rry - A^3_{33} . Rrz) Y_{roc} - A^3_{11} \cdot Mrx - Nmz \cdot a - Nmy \cdot e_m = -b_{28} \tag{28}$$

$$(- A^3_{31} . Rrx - A^3_{32} . Rry - A^3_{33} . Rrz) X_{roc} - (- A^3_{11} . Rrx - A^3_{12} . Rry - A^3_{13} . Rrz) Z_{roc} - A^3_{21} \cdot Mrx + Nmz \cdot dNm + Nmx \cdot e_m = -b_{29} \tag{29}$$

$$(- A^3_{11} . Rrx - A^3_{12} . Rry - A^3_{13} . Rrz) Y_{roc} - (- A^3_{21} . Rrx - A^3_{22} . Rry - A^3_{23} . Rrz) X_{roc} - A^3_{31} \cdot Mrx - Nmy \cdot dNm + Nmx \cdot a = -b_{30} \tag{30}$$

$$Rlx - 2A^4_{11} \cdot Rnx - 2A^4_{13} \cdot Rnz + Ncx = -b_{31} \tag{31}$$

$$Rly - 2A^4_{21} \cdot Rnx - 2A^4_{23} \cdot Rnz + Ncy = -b_{32} \tag{32}$$

$$Rlz - 2A^4_{31} \cdot Rnx - 2A^4_{33} \cdot Rnz + Ncz = -b_{33} \tag{33}$$

$$Rly \cdot Z_{lc} - Rlz \cdot Y_{lc} + Ncy \cdot Z_{o1c} - Ncz \cdot Y_{o1c} = -b_{34} \tag{34}$$

$$Rlz \cdot X_{lc} - Rlx \cdot Z_{lc} + Ncz \cdot X_{o1c} - Ncx \cdot Z_{o1c} = -b_{35} \tag{35}$$

$$Rlx \cdot Y_{lc} - Rly \cdot X_{lc} + Ncx \cdot Y_{o1c} - Ncy \cdot X_{o1c} = -b_{36} \tag{36}$$

$$Rnx + Nnx = -b_{37} \tag{37}$$

$$Rny + Nny = -b_{38} \tag{38}$$

$$Rnz + Nnz = -b_{39} \tag{39}$$

$$-Nmy \cdot e_m + Mnx = -b_{40} \tag{40}$$

$$Nmz \cdot dNn + Nmx \cdot e_m = -b_{41} \tag{41}$$

$$-Nmy \cdot dNn = -b_{42} \tag{42}$$

$$Rcx - A^5_{11} . Rnx - A^5_{12} . Rny - A^5_{13} . Rnz + Nax = -b_{43} \tag{43}$$

$$Rcy - A^5_{21} . Rnx - A^5_{22} . Rny - A^5_{23} . Rnz + Nay = -b_{44} \tag{44}$$

$$Rcz - A^5_{31} . Rnx - A^5_{32} . Rny - A^5_{33} . Rnz + Naz = -b_{45} \tag{45}$$

$$(- A^5_{21} . Rnx - A^5_{22} . Rny - A^5_{23} . Rnz)\, Z_{nc} - (- A^5_{31} . Rnx - A^5_{32} . Rny - A^5_{33} . Rnz)\, Y_{nc} - A^5_{11} \cdot Mnx - Ncy \cdot e_a = -b_{46} \tag{46}$$

$$(- A^5_{31} . Rnx - A^5_{32} . Rny - A^5_{33} . Rnz)\, X_{nc} - (- A^5_{11} . Rnx - A^5_{12} . Rny - A^5_{13} . Rnz)\, Z_{nc} - A^5_{21} \cdot Mnx + Ncx \cdot e_a + Ncz \cdot dNa = -b_{47} \tag{47}$$

$$(- A^5_{11} . Rn - A^5_{12} . Rny - A^5_{13} . Rnz)\, Y_{nc} - (- A^5_{21} . Rnx - A^5_{22} . Rny - A^5_{23} . Rnz)\, X_{nc} - A^5_{31} \cdot Mnx + Mcz - Ncy \cdot dNa = -b_{48} \tag{48}$$

$$Rhx - A^6_{11} . Rcx - A^6_{12} . Rcy - A^6_{13} . Rcz = -b_{49} \tag{49}$$

$$Rhy - A^6_{21} . Rcx - A^6_{22} . Rcy - A^6_{23} . Rcz = -b_{50} \tag{50}$$

$$Rhz - A^6_{31} . Rcx - A^6_{32} . Rcy - A^6_{33} . Rcz = -b_{51} \tag{51}$$

$$(- A^6_{21} . Rcx - A^6_{22} . Rcy - A^6_{23} . Rcz)\, Z_{codh} - (- A^6_{31} . Rcx - A^6_{32} . Rcy - A^6_{33} . Rcz)\, Y_{codh} - A^6_{13} \cdot Mcz = -b_{52} \tag{52}$$

$$(- A^6_{31} . Rcx - A^6_{32} . Rcy - A^6_{33} . Rcz)\, X_{codh} - (- A^6_{11} . Rcx - A^6_{12} . Rcy - A^6_{13} . Rcz)\, Z_{codh} - A^6_{23} \cdot Mcz = -b_{53} \tag{53}$$

$$(- A^6_{11} . Rcx - A^6_{12} . Rcy - A^6_{13} . Rcz)\, Y_{codh} - (- A^6_{21} . Rcx - A^6_{22} . Rcy - A^6_{23} . Rcz)\, X_{codh} - A^6_{33} \cdot Mcz = -b_{54} \tag{54}$$

$$Rpx - A^7_{11} . Rhx - A^7_{12} . Rhy - A^7_{13} . Rhz = -b_{55} \tag{55}$$

$$Rpy - A^7_{21} . Rhx - A^7_{22} . Rhy - A^7_{23} . Rhz = -b_{56} \tag{56}$$

$$Rpz - A^7_{31} . Rhx - A^7_{32} . Rhy - A^7_{33} . Rhz = -b_{57} \tag{57}$$

$(- A^{7}_{21} . Rhx – A^{7}_{22} . Rhy – A^{7}_{23} . Rhz) Z_{hpal} - (- A^{7}_{31} . Rhx – A^{7}_{32} . Rhy – A^{7}_{33} . Rhz) Y_{hpal} + Mpx = -b_{58}$ (58)

$(- A^{7}_{31} . Rhx – A^{7}_{32} . Rhy – A^{7}_{33} . Rhz) X_{hpal} - (- A^{7}_{11} . Rhx – A^{7}_{12} . Rhy – A^{7}_{13} . Rhz) Z_{hpal} –Mpy = -b_{59}$ (59)

$(- A^{7}_{11} . Rhx – A^{7}_{12} . Rhy – A^{7}_{13} . Rhz) Y_{codh} - (- A^{7}_{21} . Rhx – A^{7}_{22} . Rhy – A^{7}_{23} . Rhz) X_{codh} = -b_{60}$ (60)

$-Rlx – A^{8}_{11} \cdot Rpx – A^{8}_{12} \cdot Rpy – A^{9}_{11} \cdot Rpx + A^{9}_{12} \cdot Rpy + Nex – A^{10}_{11} \cdot Rc1x – A^{10}_{12} \cdot Rc1y – A^{10}_{13} \cdot Rc1z = -b_{61}$ (61)

$-Rly – A^{8}_{21} \cdot Rpx – A^{8}_{22} \cdot Rpy – A^{9}_{21} \cdot Rpx + A^{9}_{22} \cdot Rpy + Ney – A^{10}_{21} \cdot Rc1x – A^{10}_{22} \cdot Rc1y – A^{10}_{23} \cdot Rc1z = -b_{62}$ (62)

$-Rlz – A^{8}_{31} \cdot Rpx – A^{8}_{32} \cdot Rpy – A^{9}_{31} \cdot Rpx + A^{9}_{32} \cdot Rpy + Nez – A^{10}_{31} \cdot Rc1x – A^{10}_{32} \cdot Rc1y -A^{10}_{33} \cdot Rc1z = -b_{63}$ (63)

$(– A^{10}_{21} \cdot Rc1x – A^{10}_{22} \cdot Rc1y – A^{10}_{23} \cdot Rc1z) Z_{cerl} – (– A^{10}_{31} \cdot Rc1x – A^{10}_{32} \cdot Rc1y – A^{10}_{33} \cdot Rc1z) +Y_{cerl} + Ney \cdot dNe + (-A^{9}_{21} \cdot Rpx + A^{9}_{22} \cdot Rpy) Z_{pall} + (-A^{9}_{31} \cdot Rpx + A^{9}_{32} \cdot Rpy) Y_{pall} + (-A^{8}_{21} \cdot Rpx - A^{8}_{22} \cdot Rpy) Z_{pall} - (-A^{8}_{31} \cdot Rpx - A^{8}_{32} \cdot Rpy) Y_{pall} - A^{8}_{11} \cdot Mpx – A^{8}_{12} \cdot Mpy + A^{9}_{11} \cdot Mpx – A^{9}_{12} \cdot Mpy = -b_{64}$ (64)

$(– A^{10}_{31} \cdot Rc1x – A^{10}_{32} \cdot Rc1y – A^{10}_{33} \cdot Rc1z) X_{cerl} – (– A^{10}_{11} \cdot Rc1x – A^{10}_{12} \cdot Rc1y – A^{10}_{13} \cdot Rc1z) Z_{cerl} - Nex \cdot dNe - Nez \cdot e_e + (-A^{9}_{31} \cdot Rpx + A^{9}_{32} \cdot Rpy) X_{pall} - (-A^{9}_{11} \cdot Rpx + A^{9}_{12} \cdot Rpy) Z_{pall} + (-A^{8}_{31} \cdot Rpx - A^{8}_{32} \cdot Rpy) X_{pall} - (-A^{8}_{11} \cdot Rpx - A^{8}_{12} \cdot Rpy) Z_{pall} - A^{8}_{21} \cdot Mpx – A^{8}_{22} \cdot Mpy + A^{9}_{21} \cdot Mpx – A^{9}_{22} \cdot Mpy = -b_{65}$ (65)

$(– A^{10}_{11} \cdot Rc1x – A^{10}_{12} \cdot Rc1y – A^{10}_{13} \cdot Rc1z) Y_{cerl} – (– A^{10}_{21} \cdot Rc1x – A^{10}_{22} \cdot Rc1y – A^{10}_{23} \cdot Rc1z) X_{cerl} + Ney \cdot e_e - (-A^{9}_{11} \cdot Rpx + A^{9}_{12} \cdot Rpy) X_{pall} - (-A^{9}_{21} \cdot Rpx + A^{9}_{22} \cdot Rpy) X_{pall} + (-A^{8}_{11} \cdot Rpx - A^{8}_{12} \cdot Rpy) Y_{pall} - (-A^{8}_{21} \cdot Rpx - A^{8}_{22} \cdot Rpy) X_{pall} - A^{8}_{31} \cdot Mpx – A^{8}_{32} \cdot Mpy + A^{9}_{31} \cdot Mpx – A^{9}_{32} \cdot Mpy = -b_{66}$ (66)

This difficulty can be solved programming a numerical iterative method of resolution. In our case we opted by the position of a Newton-Raphson. Expressing the mathematical form as:

$[A(x)] [x] = [b]$

and outlining the iterative process of Newton-Raphson:

$[f] = [A(x)] * [x]$

$\Delta b= [b] – [f]$

$\Delta x = (G^{-1})*\Delta b$

$x = x +\Delta x$

we notice that the problem cannot be solved for not existing inverse of the Jacobiano of the matrix A(x) that is G, and it presents range 62.

This means that exist infinite solutions for the unknown vector. Obviously the real problem of a person's contact count an only balance state in a certain geometric configuration, which comes us to indicate that the biomechanical pattern outlined is not completely representative of the real human body. However a much more positive reading exists, because we can agree that we have generated a biomechanical position for a multibody system very next to the complex reality of the human body.

The solution outlined for the resolution of the problem in our case needs to eliminate four columns so the range of the Jacobiano of [A(x)] it continues being 62. It is necessary therefore to evaluate which are the groups of four possible columns to eliminate. As it is a physical fact that the 66 equations should be completed and it is a mathematical fact that all cannot be completed at the same time, we opted to find the nearest solution to all them. Analyzing the new problem:

$[A'(x)]\ [x] = [b]$

where [A'(x)] is the new matrix of coefficients eliminating the four opportune columns, we can transform the equation multiplying by the traspose of [A'(x)], obtaining on this way a new problem whose solution completes to possess a half minimum quadratic error regarding the 66 equations outlined in a principle.

$[A'(x)]^t\ [A'(x)]\ [x] = [A'(x)]^t\ [b]$

being able to write that:

$[A''(x)]\ [x] = [A'(x)]^t\ [b]$
$[f] = [A''(x)] * [x]$
$\Delta b= [A'(x)]^t\ [b] - [f]$
$\Delta x = ([G(x)]^{\wedge}\text{-}1)*\Delta b$
$x = x + \Delta x$

so we would have insured the resolution process to have a matrix [A " (x)] of coefficients whose Jacobiano [G(x)] possesses range 62. On this way the iterative method would converge like it has been commented to a solution that doesn't complete the 66 demanded equations evidently but that approaches under a half minimum quadratic error to all them. In this respect it is worthwhile to take a lot of diligence in the analysis of the problem.

To eliminate one of the columns of the matrix of coefficients supposes trivially to assume an associate final value of the unknown, it means, to eliminate the column "j" supposes to assume a final value for the unknown "Xj". This implies that it should be analyzed which unknowns can be estimated and in which condition to be able to eliminate the respective columns.

One of the positions that overcomes the difficulties of the outlined problem tries to look for the group of four eliminable columns conformed by considerable parameters. For it:

- we will be carried out in a rational way an evaluation of all those parameters that can be considerable and the conditions in which they can be it

- we will program a computational method that evaluates which of the possible groups of columns conformed by considerable parameters maintains the range of the Jacobiano of [A'(x)] = 62.

So we will have groups of columns that offer range of the Jacobiano of [A'(x)] = 62 while we eliminate considerable parameters. However it will be necessary to evaluate each one of the groups of columns for singular, since it can happen that one of the parameters is considerable under conditions in those that another cannot be it, being for it useless this group of columns in our problem. So we can understand in a generic way that is possible to find different groups of columns in function of the analyzed geometric configuration, being seen divided therefore the position of the problem.

Next we show which of the 66 parameters can be a priori considerable:

Ncy, Ncz, dNc, Npx, Npy, dNp, Ngy, Ngz, dNg, Nmx, Nmy, Ncoxy, Nnx, Nny, dNn, Nax, Nay, dNa, Ney, Nez, dNe, Mtz, Mnx

With this list of parameters we can program an executable file that evaluates which are the possible groups that we can form with this such parameters that eliminated they maintain range 62. In the first place is defined the potential group of unknownn eliminable by means of their coordinate in the vector of unknown:

E=[5,6,7,14,15,17,18,23,24,25,30,31,38,43,44,46,47,51,52,54,64,65,66]

For later on to program the iterative process of evaluation that once executed offers the following combinations of columns like potential solutions to expense of evaluating their estimability:

14 17 24 25 / 14 17 24 30 / 14 17 24 31 / 14 17 25 30 / 14 17 25 31
14 17 30 31 / 14 18 24 25 / 14 18 24 30 / 14 18 24 31 / 14 18 25 30
14 18 25 31 / 14 18 30 31 / 14 23 24 25 / 14 23 24 30 / 14 23 24 31
14 23 25 30 / 14 23 25 31 / 14 23 30 31 / 14 24 25 30 / 14 24 25 31
14 24 30 31 / 14 25 30 31 / 17 24 25 30 / 17 24 25 31 / 17 24 30 31
17 25 30 31 / 18 24 25 30 / 18 24 25 31 / 18 24 30 31 / 18 25 30 31
23 24 25 30 / 23 24 25 31 / 23 24 30 31 / 23 25 30 31 / 24 25 30 31

As can observe only 8 of the 23 considerable parameters give place to groups of eliminable columns. These parameters are (14, 17, 18, 23, 24, 25, 30, 31), or more concretely: *Npx, dNp, Mtz, Ngy, Ngz, dNg Nmx, Nmy*

Now we should analyze the estimate possibilities of each one of these 8 parameters with the purpose of knowing the conditions of good estimate, it means:

- This parameters will be able to take or don't value considerable in certain occasions
- This occasions will only be function of the geometric configuration of the position of rest, and never of the biotype or the percentile.
- The equation of estimate of each parameter should be found in function of the angular parameters of corporal positioning.
- It should be defined the groups of columns to eliminate for each certain combination of angles with the purpose of elaborating a map of groups in function of the corporal position.

Next a very brief summary of the estimate conditions will be exposed for each one of these 8 parameters for separate:

Npx = 14 = longitudinal Friction in the support of the foot

$\alpha_m+\alpha_r+\alpha_t \approx 0 \Rightarrow Npx = 0$
$\alpha_m+\alpha_r+\alpha_t \approx \pi/2 \Rightarrow Npx = 0$

dNp = 17 = Position of the center of pressures of the contact of the foot

$\alpha_m+\alpha_r+\alpha_t \approx 0$ => dNp can be considered by means of results of the system of acquisition of data of pressures
$\alpha_m+\alpha_r+\alpha_t =\ldots$ => $dNp= f(\alpha_m+\alpha_r+\alpha_{t=0})$ is considered again by means of systems of acquisition of data.

Mtz = 18 = Even of reaction in the ankle

$((\phi_m \approx 0 \approx \beta_m) \Rightarrow \forall\ (\alpha_m, \alpha_r, \alpha_t)\ Mtz \approx 0$
$(\phi_m \neq 0) \cap (\alpha_m \approx 0 \approx \beta_m) \Rightarrow \forall\ (\alpha_r, \alpha_t)\ Mtz \approx 0$

Ngy = 23 = traverse Friction in the twin's support

$(\phi_m \approx 0 \approx \beta_m) \Rightarrow \forall\ (\alpha_m, \alpha_r, \alpha_t)\ Ngy \approx 0$
$(\phi_m \neq 0) \cap (\alpha_m \approx 0 \approx \beta_m) \Rightarrow \forall\ (\alpha_r, \alpha_t)\ Ngy \approx 0$

Ngz = 24 = traverse Friction in the twin's support

It is fulfilled total generality since this friction force affects to the pant, being able to prove that in most of the cases it doesn't affect to the leg.

dNg = 25 = Position of the center of pressures of the twin's contact
$dNg = f(\alpha_m, \alpha_r, \alpha_e)$dependence function that can be considered by means of the use of systems of acquisition of data of pressure.

Nmx = 30 = longitudinal Friction in the support of the thigh

Nmx = 0 ⇔ μ=0 being m the friction coefficient

Nmy = 31 = traverse Friction in the support of the thigh

$(\phi_m \approx 0 \approx \beta_m) \Rightarrow \forall\, (\alpha_m, \alpha_r, \alpha_t)$ Nmy ≈ 0
$(\phi_m \neq 0) \cap (\alpha_m \approx 0 \approx \beta_m) \Rightarrow \forall\, (\alpha_r, \alpha_t)$ Nmy ≈ 0
$(\alpha_m \approx 0) \Rightarrow$ Nmy ≈ 0

- Ngz = X(24) = 0 ⇒ we can assume with guarantees since this force affects to the slip of the pant on the skin

- *dNp*= X(17), *dNg* = X(25) ⇒ is possible to estimate them experimentally

- Mtz = X(18) = 0, Ngy = X(23) = considerable 0 Alone Þ in simple positions of sedence, and like one can observe in the groups in a separate way, never simultaneously. $(\phi_m \approx 0 \approx \beta_m)$ $(\phi_m \neq 0, \alpha_m \approx 0 \approx \beta_m)$. Their estimate has priority over X(17) and X(25).

- Nmy = X(31) = 0 Considerable Þ in simple positions of sedence. $(\alpha_m \approx 0)$, $(\phi_m \approx 0 \approx \beta_m)$. whenever X(23) or X(18) are is acceptable it is assumed.

- Npx = X(14) = 0 Considerable Þ in very limited positions of sedence. For it Npx = $f(\alpha_m+\alpha_r+\alpha_{t=0})$ that one obtains experimentally.

Finally we have the parameters (14, 17, 18, 23, 24, 25, 30), with those the combinations decrease to:

14 17 24 25 / 14 17 24 31 / 14 17 25 31 / 14 18 24 25 / 14 18 24 31
14 18 25 31 / 14 23 24 25 / 14 23 24 31 / 14 23 25 31 / 14 24 25 31
17 24 25 31 / 18 24 25 31 / 23 24 25 31

However it will be necessary to impose the values of a limit somehow starting from which am, bm and fm stop to be considered approximately null for the estimates of the unknown ones. In this respect it is easy to understand that this frontier of values doesn't have to be of constant value, if not that it can be therefore function of the combinations of corporal angles that define the position, in a generic way and without knowing this functions we can express:

$\alpha_m \approx 0 \Leftrightarrow \alpha_m < \alpha_o\,(\alpha_m,\alpha_r,\alpha_e) = \alpha_o$
$\beta_m \approx 0 \Leftrightarrow \beta_m < \beta_o\,(\alpha_m,\alpha_r,\alpha_e) = \beta_o$
$\phi_m \approx 0 \Leftrightarrow \phi_m < \phi_o\,(\alpha_m,\alpha_r,\alpha_e) = \phi_o$

Having outlined the dependence of the values frontier in function of the three angles am,ar,ae has their rational reasons that here don't clear up to have no importance. The problem increases its difficulty when exist combinations of unknowns in certain such groups that their estimates depend from the vicinity to the nullity or not of oneself parameter, which would converge to the possible existence of two different frontiers, being necessary to choose the smaller one for nullity or the biggest in the other case.

Conjugating all the previous information we arrive to the following groups of columns separation in function of the geometric configuration to study:

1- $[\alpha_m \in (0, \alpha_o) \cap \beta_m \in (\beta_o, \pi/2) \cap \phi_m \in (0, \pi/2)] \Rightarrow$ (17, 24, 25, 31) = A

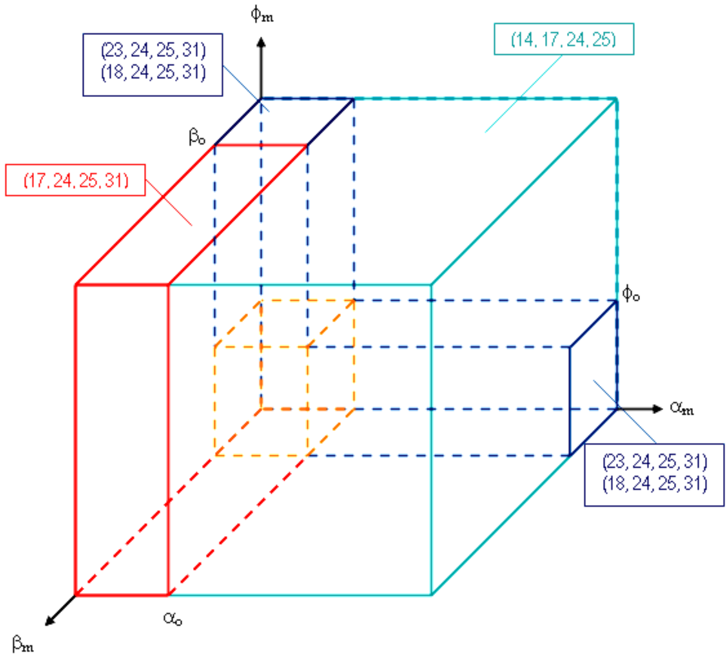
2- $[\alpha_m \in (0, \alpha_o) \cap \beta_m \in (0, \beta_o) \cap \phi_m \in (\phi_o, \pi/2)] \Rightarrow$ (23, 24, 25, 31) = B (18, 24, 25, 31) = C

3- $[\alpha_m \in (\alpha_o, \pi/2) \cap \beta_m \in (0, \beta_o) \cap \phi_m \in (\phi_o, \pi/2)] \Rightarrow$ (23, 24, 25, 31) = B (18, 24, 25, 31) = C

4- $[\alpha_m \in (0, \alpha_o) \cap \beta_m \in (0, \beta_o) \cap \phi_m \in (0, \phi_o)] \Rightarrow$ (17, 24, 25, 31) = A (23, 24, 25, 31) = B (18, 24, 25, 31) = C

5- Demás casos $\Rightarrow$ (14, 17, 24, 25) = D

that in a graphic way under the hypothesis that ao, bo, fo is constant "(am,ar,ae) it would turn out to be:



The previous graph represents the different groups of columns that should be eliminated for the resolution of the problem in function of the combination of angles characteristic am, bm and low fm the hypothesis of constant frontiers in the definition of the fields of consideration of nullity of the angular parameters implied in the estimate ao, bo and fo.

With everything it has been possible to program an executable file such that introducing the Antropometric Percentile of the person, the IMC and the corporal position to analyze, offers in an almost immediate way the value of the superficial and articular reactions, fundamental base of the healthy conditions and existent comfort for each person in each sedence position.