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Design of novel adaptive structures based on bistable composites

W. Hufenbach, M. Gude, A. Czulak

Institut für Leichtbau und Kunststofftechnik (ILK), Technische Universität Dresden
01062 Dresden, Germany

1. INTRODUCTION

During the manufacture of multilayered fibre- and textile-reinforced composites with variable fibre orientations, residual stress states arise due to the directional expansion of the unidirectionally (UD) reinforced single layers or the basic textile layers. Dependent on the laminate lay-up and the textile architecture, these inhomogeneous residual stresses, which are primarily caused by thermal effects, moisture absorption and chemical shrinkage, can lead to large mono- and multistable out-of-plane deformations in the case of unsymmetric laminates (Fig. 1).

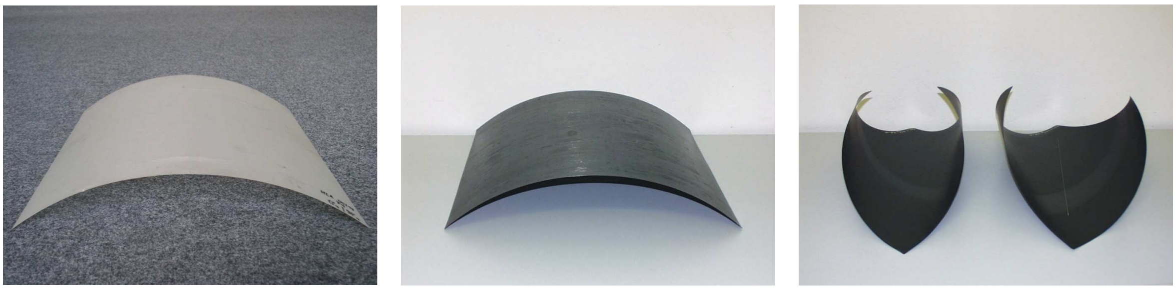

Fig. 1: Multistable deformation states of unsymmetric composites.

Instead of avoiding these laminate’s curvatures and the belonging multistable deformation states, they can be advantageously used for technical applications such as novel adaptive structures. In order to adjust the laminate deformations to technical requirements, a dimensioning tool based on a modified stability analysis has been developed and experimentally verified. With the help of the theoretical model, adaptive prototypes of multistable composites with integrated smart alloys have been designed and afterwards manufactured and tested.

2. NON-LINEAR DEFORMATION THEORY OF MULTISTABLE COMPOSITES

For unsymmetric laminates, the hygro-thermally and chemically caused directional deformations of the UD single layers result in large out-of-plane deformations. To take into account the large multistable deformations, which are often many times over the laminate thickness, the linear strain-displacement relations must be extended by non-linear terms (see e.g. [1-4]):

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 - z\frac{\partial^2 w}{\partial x^2} = \frac{\partial u^0}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2},\\
\varepsilon_y &= \varepsilon_y^0 - z\frac{\partial^2 w}{\partial y^2} = \frac{\partial v^0}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2},\\
\varepsilon_{xy} &= \varepsilon_{xy}^0 - z\frac{\partial^2 w}{\partial x \partial y} = \frac{1}{2}\left(\frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) - z\frac{\partial^2 w}{\partial x \partial y},
\end{aligned} \tag{1}$$

where the index 0 refers to the laminate reference plane.

The developed dimensioning tool for unsymmetric composites is based on the principle of minimising the total potential energy, which is given here by

$$\Pi = \int_V \left(\frac{1}{2}\overline{Q}_{ij}\varepsilon_i\varepsilon_j - \eta_{Ti}\varepsilon_i\Delta T - \eta_{Mi}\varepsilon_i\Delta M - \eta_{Si}\varepsilon_i\right) dV \tag{2}$$

with (i, j = 1, 2, 6), where the $\overline{Q}_{ij}$ are the reduced transformed stiffnesses, ΔT and ΔM are the differences in temperature and relative media concentration with respect to the reference state and η_{Ti}, η_{Mi}, η_{Si} are related to the elastic constants and to the thermal expansion coefficients α_j ($\eta_{Ti} = \overline{Q}_{ij}\alpha_j$), the swelling coefficients β_j ($\eta_{Mi} = \overline{Q}_{ij}\beta_j$) and the shrinkage s_j ($\eta_{Si} = \overline{Q}_{ij}s_j$) respectively [5].

Based on the total potential energy, the Rayleigh-Ritz method is applied to obtain approximate solutions for the resulting displacement fields. Therefore, general approaches in the form of polynomials are used dependent on the laminate lay-up and textile structure.

For a square cross-ply $[0_n/90_n]$ laminate the occurring basic shapes are illustrated in Fig. 2. Starting from the plane shape, which is considered as the reference state (Fig. 1a), the residual stresses lead – dependent on the laminate dimensions and non-mechanical loads – to a saddle shape (b) or to either of the two stable cylindrical shapes (c, d) (see also [5]). For $[0_n/90_m]$ laminates with n << m or n >> m however only one stable cylindrical shape occurs.

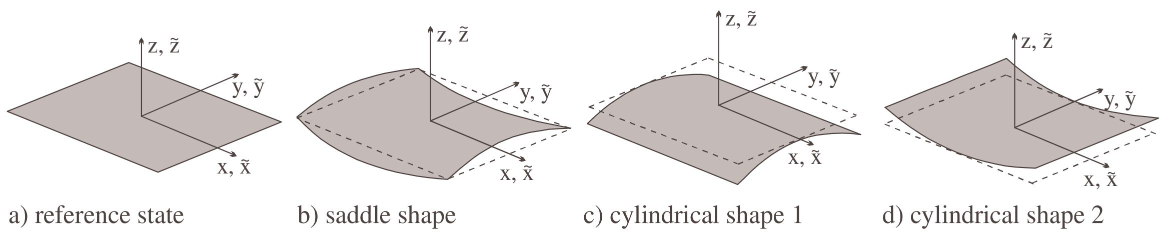

Fig. 2: Basic shapes of square $[0_n/90_n]$ laminates; a) reference state at elevated curing temperature, b) - d) saddle and cylinder shapes at room temperature

Based on adapted displacement approaches with unknown Ritz coefficients the principle of the minimum total potential energy in combination with the Rayleigh-Ritz method leads to a non-linear equation system, which is derived form the first variation according to

$$\delta\Pi = \frac{\partial \Pi}{\partial a_k}\delta a_k + \frac{\partial \Pi}{\partial b_k}\delta b_k \equiv 0 \tag{3}$$

with the unknown Ritz coefficients a_k, b_k (k = 0, 1, 3).

Dependent on the laminate lay-up and the laminate size, more than one solution can be obtained, which describe stable, indifferent and instable equilibrium states. Thus, these solutions have to be checked for their stability by means of $\delta^2\Pi$, which has to be positive for a stable deformation state.

3. ANALYSIS OF EXEMPLARY UNSYMMETRIC COMPOSITES

Applying the above mentioned theory and the developed simulation tool, the solutions expressed in terms of the curvatures of an exemplary square $[0_2/90_2]$ CFRP laminate are shown in Fig. 3 dependent on the laminate edge length L. Branch AB shows the laminate curvature of the stable saddle shape ($\kappa_x = a_0 = -b_0 = -\kappa_y$), which occurs in the case of small laminates. The critical edge length for the saddle shape is where the saddle shape becomes unstable, which defines the bifurcation point (B). After this bifurcation point, the saddle shape occurs only theoretically as an unstable equilibrium state but not in reality, which is indicated by the dashed line BC. Instead of the stable saddle shape, now two equivalent stable (bistable) solutions are calculated (BD and BE). Branch BD represents the curvature a_0 of the cylindrical shape in Fig. 2d and the curvature $-b_0$ of the cylindrical shape in Fig. 2c. Branch BE represents the secondary curvature $-b_0$ of the cylindrical shape in Fig. 2d and the secondary curvature a_0 of the cylindrical shape in Fig. 2c, which asymptotically approach zero with an increasing edge length L.

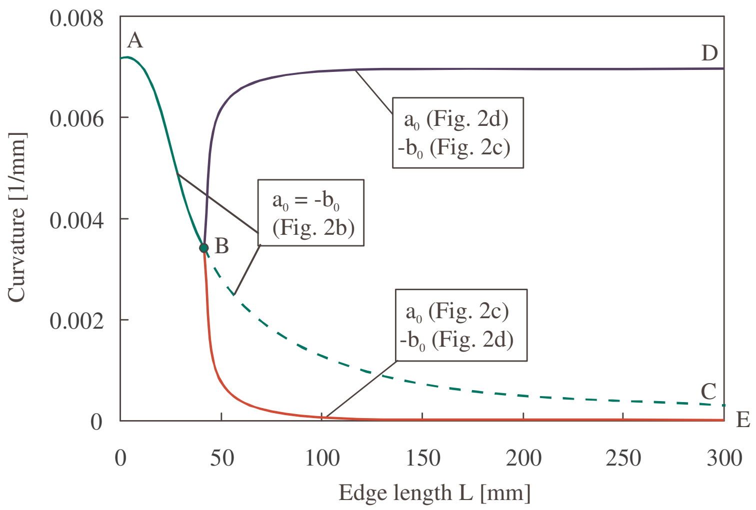

Fig. 1: Curvatures of a $[0_2/90_2]$ CFRP-laminate dependent on laminate edge length L.

4. ADAPTIVE MULTISTABLE STRUCTURES

The combination of the described bistable composites with actuators enables the design of novel adaptive structures. By advantageously using the existence of different stable deformation states, large adaptive deformations and forces can be realised by a short electrical impulse in contrast to conventional adaptive structures, which need a continuous adaptive support to realise only small deflections [5, 6]. Thus, the different stable deformation states of a cross-ply laminate for instance can be changed by snap-through from cylindrical shape 1 to cylindrical shape 2 (Fig. 4).

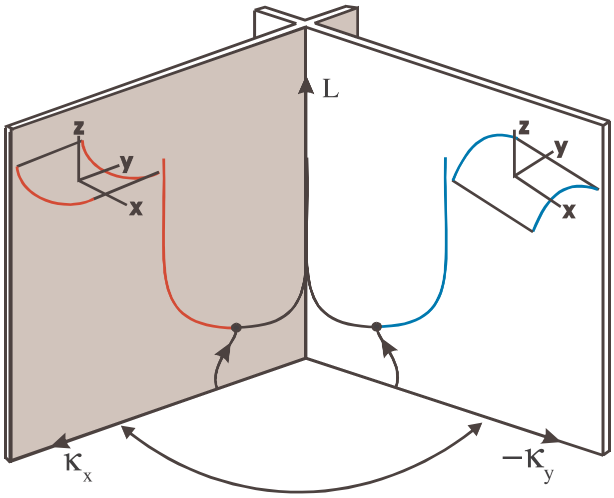

Fig. 4: Snap-through between stable deformation states.

The novel adaptive structures consist of passive composite layers which are arranged in that way that a bi-stable laminate results due to residual stresses. The second components are smart materials, which are embedded in the laminate as it is shown for cross-ply laminates in Fig. 5.

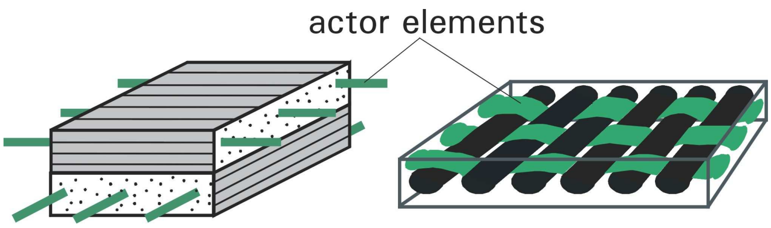

Fig. 5: Integration of smart alloys in multistable fibre- and textile reinforced composite.

The necessary mechanical and adaptive properties of the composite and the adaptive components can be calculated based on the developed semi-analytical method. To initiate the snap-through, adaptive forces and moments have to be superposed to the residual stress state according to the extended structural law

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{bmatrix} - \begin{bmatrix} \mathbf{N}_\mathrm{T} \\ \mathbf{M}_\mathrm{T} \end{bmatrix} - \begin{bmatrix} \mathbf{N}_\mathrm{M} \\ \mathbf{M}_\mathrm{M} \end{bmatrix} - \begin{bmatrix} \mathbf{N}_\mathrm{A} \\ \mathbf{M}_\mathrm{A} \end{bmatrix} \tag{4}$$

with the extension, coupling and bending stiffness matrices A, B, D and the resulting mechanical, thermal, medial and adaptive forces (N, N_T, N_M, N_A) and moments (M, M_T, M_M, M_A).

Based on the theoretical investigations, a smart prototype of multistable composite made of CFRP have been designed and manufactured. Load adapted smart alloys (NiTi wires) were fixed on the top and the bottom of the cylindrical curved structure and integrated into an electrical circuit (Fig. 6). The snap-through effect was successfully initiated by the actors and the predicted actor forces and deformation were in good agreement with the theoretical results.

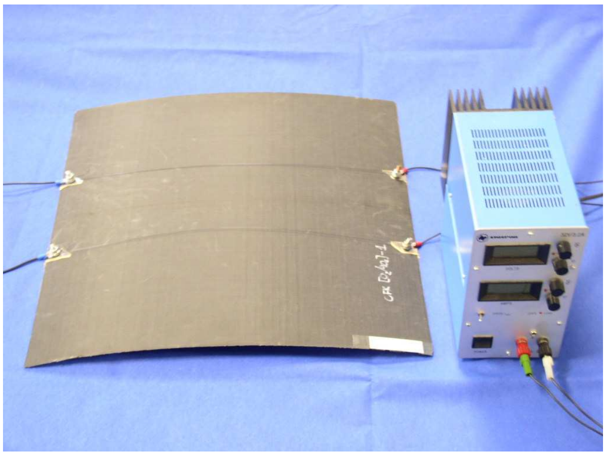

Fig. 6: Prototype of an adaptive multistable composite.

The new adaptive structures can be adjusted to individual technical requirements by designing a load-adapted composite structure. For instance, besides switching between two equivalent cylindrical shapes in the case of $[0_n/90_n]$ laminates it can also be switched between two non-equivalent deformation states of $[0_n/90_m]$ laminates or between bi-stable deformation states of general angle-ply laminates [7]. Thus, the novel adaptive structures offer a high potential for transfer of forces or for usage as components in further adjusting devices, mechanisms, valve flaps or active membranes.

5. CONCLUSIONS

The residual stress field, which builds up during the manufacturing process of multilayered fibre-reinforced composites with variable fibre orientations can be purposefully used to design unsymmetric composites with defined deformation states and curvatures. Therefore, a modified stability analysis has been developed, which enables the prediction of the resulting monostable and multistable deformation states due to thermal effects, moisture absorption and chemical shrinkage. Furthermore the theoretical model for the calculation of the multistable deformation states was extended for composites with integrated actor elements. It has been shown that the new calculation method enables a detailed assessment of the design parameters and serves for the efficient construction of multilayered composites and novel adaptive structures.

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