



12th INTERNATIONAL SCIENTIFIC CONFERENCE  
ACHIEVEMENTS IN MECHANICAL & MATERIALS ENGINEERING

Boundary element stress analysis for bi-metallic dies in pressure die casting

M. T. Alonso Rasgado, K. Davey, L. D. Clark and S. Hinduja

Department of Mechanical, Aerospace and Manufacturing Engineering  
UMIST, Manchester

The rate of heat extraction during the pressure die casting process is central to both the quality and the cost of the finished castings. Energy extraction rates in traditional steel dies are principally dictated by the cooling arrangement but also by the resistance path offered by the die. It has recently been shown that modification to cooling channel shapes and position can enhance sub-cooled nucleate boiling and substantially increase energy-extraction rates. Reducing the thermal resistance of the coolant boundary layer means that a significant proportion of the thermal resistance path becomes attributable to the die steel. Replacing the steel with copper can greatly improve heat extraction rates.

This paper is concerned with the development of an efficient structural model for the analysis of bi-metallic copper-alloy steel dies. A cyclic boundary element stress model has been developed so that stress levels can be predicted and examined in both the copper and steel parts of the die. The stress model is based on the 3-D thermoelastic boundary element method and produces stresses and deformation due to both mechanical and thermal loads on the system. The collocation based boundary element method is used for the prediction of transient stress fields over a thermally stabilised casting cycle. A novel approach called the simplex method is used to evaluate domain integrals that appear in the boundary integral formulation due to the presence of a transiently varying thermal field. This method involves meshing the domain with tetrahedral elements and then applying recursive radial integration to these tetrahedral elements. Repetitive application of the recursive scheme effectively reduces the initial volume integral to line integrals. Excellent accuracy is obtained with this method. In order to validate the model predicted strain fields are compared with strain gauge measurements obtained on a purpose built rig designed to be representative of the casting process and on a production die.

## 1. INTRODUCTION

Pressure die casting is an important industrial process used for the mass production of complex components. Dies are usually manufactured from tool steel, hardened to increase longevity as they are required to withstand high mechanical and thermal loads associated with the process and typically must be capable of producing in excess of 100,000 components. Dies manufactured from copper alloy, combined with a thin layer of tool steel for strength purposes, have the potential for delivering much higher rates of heat extraction. The boundary element model described in this paper incorporates the alloyed steel layer and is capable of

predicting transient stresses for the copper dies and alloyed steel layer throughout the casting cycle due to the applied thermal and mechanical loads. In the case of copper-steel alloy dies transient temperature variations are restricted to a narrow area around the die cavity surface of the steel layer. The mechanical loads applied to the die are primarily due to the injection pressure and the die clamping forces; the die interfaces are also subjected to a pressure that balances any surplus clamping force. The copper alloy part of the die and the steel layer are considered as two separate domains, coupled via suitable boundary conditions on the interfaces between the two. The coupling is achieved by applying high interfacial stiffness coefficients on the interfaces between the copper alloy part and steel layer, with the stiffness being sufficiently high to ensure that the displacements of corresponding points on the die and layer interface match.

## 2. THERMOELASTIC BOUNDARY ELEMENT MODEL

In this section a three-dimensional thermoelastic BE model used for predicting die deformation and stress levels in the high pressure die casting process is described. This boundary element model utilises the predicted thermal behaviour of the die blocks and component together with the applied mechanical loads within the casting process. The mechanical loads applied to the die are primarily due to the injection pressure and the die clamping forces as shown in Figure 1. The copper-alloyed domain and the steel layer are considered as two separate domains coupled by means of interfacial springs of high stiffness to ensure that the displacements of corresponding points on the interface match (see Figure 1). A fully discretised form for the thermoelastic boundary integral equation is

$$c_{\ell k}(x)u_k(x) + \sum_{n=1}^{N^k} \sum_{m=1}^{M^n} u_k^m \int_{\Delta_n^k} p_{\ell k}^*(x, y) \Theta^m \Gamma(y) - \sum_{n=1}^{N^k} \sum_{m=1}^{M^n} p_k^m \int_{\Delta_n^k} u_{\ell k}^*(x, y) \Theta^m d\Gamma(y) = \\ + \gamma \sum_{n=1}^{N^k} \sum_{m=1}^{M^n} T^m \int_{\Delta_n^k} u_{\ell k}^*(x, y) \Theta^m n_k d\Gamma(y) - \gamma \int_{\Omega^k} u_{\ell k}^*(x, y) T(y) d\Omega(z) \quad (1)$$

where;

$N^k$  - the number of elements,

$M^n$  - the number of nodes on the element,

$\Theta^m$  - the shape function, and where the superscript m indicates the nodal values for displacement, traction, temperature and heat flux,

$\Delta_n^k$  - refers to the  $n^{\text{th}}$  surface element on the  $k^{\text{th}}$  domain.

One difficulty with the numerical implementation is the evaluation of the domain integral appearing on the right hand side of equation (1) involving the transient temperature field, which typically decays rapidly from the die cavity surface into the die, extending just 3-4 mm inwards. The new method, employed to deal with this problem is discussed in section 3. Linear isoparametric or quadratic subparametric triangular elements are employed to represent the surfaces. Substituting the boundary conditions described in section 2.1 into equation (1) and taking each node in turn as a source point yields a set of linear equations, the solution of which, gives the displacement and traction at nodes on the boundary. From these results surface stress can be obtained.

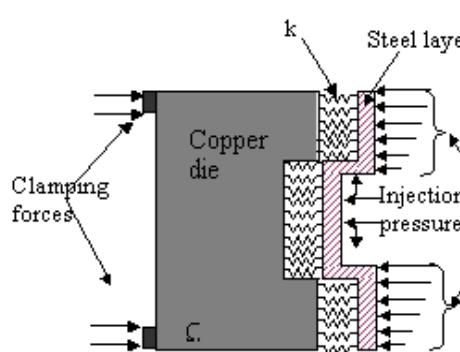
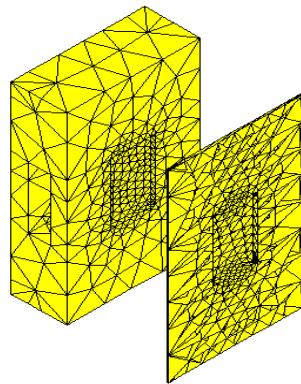


Figure 1. Mechanical loads and bonded spring on a copper die and steel layer.



A 3-D boundary element model is used to predict the transient thermal behaviour of the dies [1]. In reference [2] it is demonstrated that the majority of the die deformation is due to the applied temperature field.

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## 2.1. Steady State Boundary Conditions

This boundary element model utilises the predicted thermal behaviour of the die blocks and component together with the applied mechanical loads within the casting process. The boundary conditions on the various surfaces of a die block are as follows: *Cavity surface  $\Gamma^C$* : Traction,  $p = -P_c n$ , where  $P_c$  is the injection pressure. *Die interface  $\Gamma^{SS}$* : Traction,  $p = -P_D n$ , where  $P_D$  is the interface pressure. *Cooling channel surface  $\Gamma^W$* :  $p \approx 0$ . *External surface  $\Gamma^E$* :  $p = 0$ . *Surface connected to sliders.  $\Gamma^S$* :  $u_k = 0$ , where  $k$  depends on the constraints of the slider.

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### 3. SIMPLEX METHOD

In die casting it is recognised that the transient temperature field decays rapidly from the die cavity surface into the die, typically extending just 3-4 mm inwards. Application of the thermoelastic BEM to die casting dies necessitates the evaluation of a domain integral involving this transient temperature field. In the case of copper-steel alloy dies, for typical casting process conditions, only the steel layer generates a domain integral as the transient area is restricted to the steel layer, with no significant penetration into the copper block. The proposed method involves recursive radial integration on simplexes and is used to transform the singular domain integral over a volume element to a non-singular integral over a line, which can be evaluated to high accuracy using standard numerical quadrature. The domain integral present in the thermoelastostatic BEM can be represented in the following form,  $\int_{\Omega} u_{lk,k}^* T dV$ , where  $T$  is temperature,  $u_{lk,k}^* = C(x_\ell - x_\ell^I)/\rho^3$ , where  $C$  is related to material constants. Let  $\Omega$  be a domain of dimension 3 and consider discretisation with 3-simplexes (tetrahedral elements). Over a 3-simplex the temperature field is approximated by a linear polynomial of the form  $(a_0^j + b_m^j x_m)T_j$ , where summation over  $j$  is from 1 to 4 and  $m$  is from 1 to 3, with  $a_0^j$  and  $b_m^j$  being constants. A two-stage recursion process reduces this volume integral to a line integral [3]. The first stage of recursive integration gives

$$\begin{aligned} & \int_{S_2^k - \Gamma_{e_2}^k} \frac{1}{r_3^3} \int_{S_2^k - \Gamma_{e_2}^k} \frac{x_\ell - x_\ell^I}{r_3^3} (a_0^j + b_m^j x_m) \rho_3^2 d\rho_3 d\Gamma_2 = \\ & \left( (a_0^j + b_m^j x_m^I) \int_{S_2^k - \Gamma_{e_2}^k} \frac{x_\ell - x_\ell^I}{r_3^3} d\Gamma_2 + \frac{b_m^j}{2} \int_{S_2^k - \Gamma_{e_2}^k} \frac{(x_\ell - x_\ell^I)(x_m - x_m^I)}{r_3^3} d\Gamma_2 \right) \end{aligned} \quad (2)$$

where

$k$  is summed from 1 to 4.

A second stage of recursion applied to the first integral (say) on the right hand side of equation (2) gives

$$\begin{aligned} & \int_{S_2^k - \Gamma_{e_2}^k} \frac{x_\ell - x_\ell^I}{r_3^3} d\Gamma_2 = \\ & r_2^k \cdot \underline{n}_2^k \left( - \int_{S_1^k - \Gamma_{e_1}^k} \frac{x_\ell - x_\ell^{II}}{r_2 r_3} d\Gamma_1 + \int_{S_1^k - \Gamma_{e_1}^k} \frac{x_\ell - x_\ell^{II}}{r_2^2} \ln(r_2 + r_3) d\Gamma_1 + \int_{S_1^k - \Gamma_{e_1}^k} \frac{x_\ell^I + x_\ell^{II}}{c_{II}^2 r_2 r_3} d\Gamma_1 \right) \end{aligned} \quad (3)$$

where:

$k$  is summed from 1 to 3

$$c_{II} = \| \underline{x}^{II} - \underline{x}^I \|_2.$$

Inspection of equation (3) reveals that the remaining integrals are line integrals; these can be evaluated to high accuracy numerically [3] using standard quadrature techniques.

#### 4. EXPERIMENTAL TEST AND PREDICTIONS

The die depicted in Figure 2 was utilised in tests on an experimental rig used to simulate the die casting process. The rig was designed to reproduce the transient heat transfer mechanisms and induced stresses due to the transient thermal loads involved in die casting whilst avoiding the hazardous features of the real process. Heat is applied to the die cavity surface in a cyclic manner simulating the period when the melt is contained within the die following injection and the period following ejection of the casting when the dies are open to air. Stress results were recovered from strain measurements obtained from three strain gauges attached at various locations on the experimental die as shown in Figure 3 (a). Fibre reinforced plastic type strain gauges were employed. Ideally a temperature compensation of zero is required in order to measure thermal stress. Therefore gauges with a thermal expansion of 1.0 ppm/C were chosen. Gauges 1 and 3 measured strain in the x-direction, whilst gauge 2 measured strain in the y-direction. The thermoelastic BE model described in this paper was employed to predict deformations and stresses in the experimental die. Four casting cycles were analysed with twenty time steps being employed per cycle. The predictions obtained from the model were compared against the experimental results. Figure 3 (b) shows a comparison between predicted and measured stresses at the three locations on the die surfaces over a typical casting cycle. The shape of the plots is generally the same for all three positions. There is a sharp increase in stress close to the start of the cycle corresponding to injection when die temperatures increase rapidly. The stress continues to rise until the point at which the ejection of the casting from the dies is simulated, 0.5 seconds in the plots, after which stress falls during the period that represents the portion of the cycle for which the dies are open to air. Further inspection of Figure 3(b) reveals that in general reasonable agreement is obtained between predicted and measured stress. The main objective in modelling the thermoelastic behaviour of the copper-steel alloy dies is to obtain a tool that enables the structural integrity of the dies to be investigated prior to the manufacture and deployment of a die into production. Of particular interest are the stresses on the interface between the copper die block and the sprayed steel layer as this can affect the bonding of the two metals. It was found that static stress at the interface is predominantly compressive, which is an aid to maintaining the attachment of the steel layer to the copper die block.

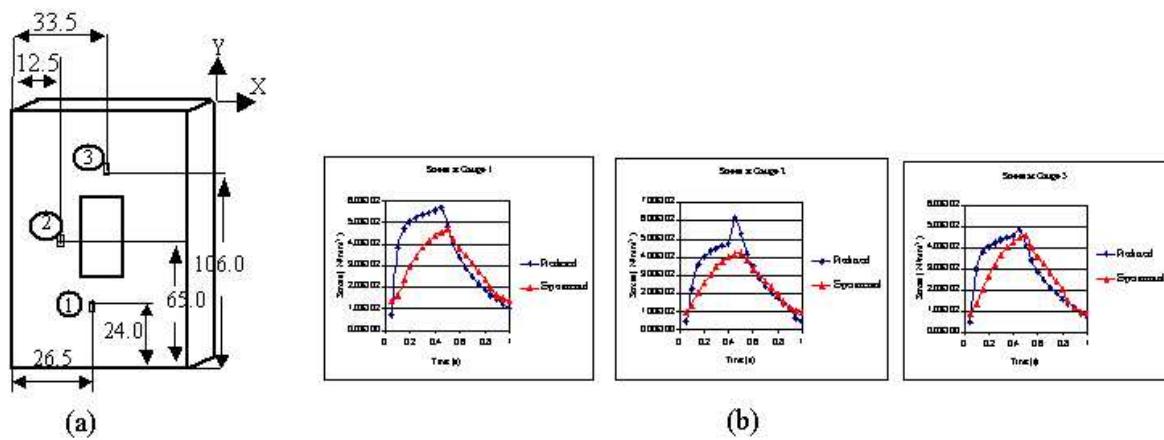


Figure 3. Strain gauge positions and predicted - experimental stress values on the die block

## 5. CONCLUSIONS

In this paper an effective numerical methodology is established founded on the boundary element method for the analysis of copper-alloyed steel composite dies. The stress model described has been shown to provide reasonable accuracy.

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