Achievements in Mechanical and Materials Engineering
COMMENT Worldwide Congress on Materials and Manufacturing Engineering and Technology
16th - 19th May 2005
Gliwice-Wisla, Poland
COMMITTEE OF MATERIALS SCIENCE OF THE POLISH ACADEMY OF SCIENCES, KATOWICE, POLAND
INSTITUTE OF ENGINEERING MATERIALS AND BIOMATERIALS OF THE SILESIAN UNIVERSITY OF TECHNOLOGY, GLIWICE, POLAND
ASSOCIATION OF THE ALUMNI OF THE SILESIAN UNIVERSITY OF TECHNOLOGY, MATERIALS ENGINEERING CIRCLE, GLIWICE, POLAND
13th INTERNATIONAL SCIENTIFIC CONFERENCE
ON ACHIEVEMENTS IN MECHANICAL AND MATERIALS ENGINEERING

Failure mode analysis using axiomatic design and non-probabilistic information

M. Pappalardo, A. Naddeo

Dept. of Mechanical Engineering, University of Salerno, Via Ponte don Melillo, 84084, Fisciano (SA), Italy

Abstract: In this paper the axioms, of Axiomatic Design, are extended to the non-probabilistic and repetitive events. The idea of information, in the classic theories of Fisher and Wiener-Shannon, is a measure only of probabilistic and repetitive events and can be extended the non-probabilistic and repetitiveness events. On the basis of Laplace's Principle of insufficient knowledge, the *MaxInf* principle is defined for choose solutions in absence of knowledge. In this paper the value of information, as a measure of equality of data among a set of values, is applied in Axiomatic Framework for data analysis in such cases in which the number of Functional Requirements (FR_s) is greater than the Design Parameter's (DP_s) one.

As example is studied an application on potential failure mechanisms in which the number of DPs is lower then the number of FR_s, and the coupled design cannot be satisfied.

Keywords: Axiomatic Design, Non-Probabilistic Information, MaxInf, Entropy, Failure mode Analysis

1. INTRODUCTION

The *distance* between two probability distribution **p** and **q** is given by the distance *D*(**p:q**). If **q** *is an priori distribution*, then to select the distribution **p** *closeness to* **q** is needed**.** For satisfying the constraints of probability distribution it's possible to use the measure of *cross-entropy* developed by Kullbach and Leibler [3]

$$D(\mathbf{p:q}) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad (1)$$

D(**p:q**) is the *distance* of the a priori distribution **q** from the distribution **p**. In order to optimise a distribution it's possible to use the minimum cross entropy principle: *From all probability distribution satisfying given constraint we must choose the distribution* ***p*** *that minimise the measure* D(**p:q**). This is the well know *MinEnt* principle. If the a priori distribution is of maximum uncertainty, $\mathbf{q} = \underbrace{1/n, \ldots, 1/n}_{n}$ then

$$D(\mathbf{p},\mathbf{q}) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{1/n} = \ln(n) - \left(-\sum_{i=1}^{n} p_i \ln p_i \right) \tag{2}$$

where

$S(\mathrm{p}) = -\sum_{i=1}^{n} p_i \ln p_i$ *is the Shannon's entropy*. Therefore, when the a priori distribution **q** has the maximum uncertainty, for minimising D(**p:q**) it's possible to choose values of **p** *maximising entropy S(***p***)*. This is the *Jaynes' maximum Entropy principle (MaxEnt)* [1].

Jaynes, the principal proponent of *MaxEnt Principle* in axiomatic way, suggests that in all probability distribution, when there's *only* the constraint that $\{p_i \geq 0\}$ and $\sum_i p_i = 1$, it's possible to choose that one has maximum entropy. The use of probability distribution with less then maximum entropy implies the use of additional information [3].

2. AXIOMS OF NEW THEORY OF INFORMATION (EXTENDED THEORY)

The idea of information, in the theories of Fisher and Wiener-Shannon [2], is a measure only on probabilistic and repetitiveness events. The idea of information is broader than the probability and the axioms of Wiener–Shannon can be extended to the non-probabilistic and repetitiveness events [10].

2. 1. New Theory of Information in Metric Space

Using the information axioms it is possible to develop models for information very useful in applications. For every event $A \in \Im$ it's possible to have a measure of information using the mathematical expression:

$$J(A) = \frac{1}{d(A)} \tag{3}$$

This definition of information has a natural application in metric space [7]. The information can be evaluated by the probability and by the non-probabilistic measures of diameters. So it's possible to have the measure of information from non-probabilistic data.

2. 2. The Extended Principle of Max-Information (MaxInf)

It is possible to define a new principle on basis of the New Theory of Information. On the analogy of *MaxEnt principle*, the name is *Max Information Principle* (*MaxInf*) [6]. In the New Theory of Information, instead of probability, it is possible to utilize of a finite number of appropriate proportion subject to a set of constraints that add up to one. In observance of the Axioms, let $d_1, d_2, \ldots, d_n$ be *n* non-negative real numbers, let [3]

$$\begin{cases} \sum_{i=1}^{n} d_i \neq 0 \quad \rho_i = \dfrac{d_i}{d_1 + d_2 + \ldots + d_n} \\ \sum_{i=1}^{n} \rho_i = 1 \qquad \rho_i \geq 0 \ \forall \mathrm{i} \end{cases} \tag{4}$$

It's possible to use the measure of information the relation

$$J(\rho) = J(\rho_1, \quad \rho_2, ..., \quad \rho_n) = -\sum_{i=1}^{n} \rho_i \ln \rho_i \tag{5}$$

So that: *J(ρ)* is maximum when $\rho_1 = \rho_2 = ... = \rho_n$**,** *J(ρ)* is minimum when: $\forall i$ only one number is $\neq$ zero; in metric space, using Euclidean's distances the information may be

$$J = -\frac{\sum_{ij} |d(x_i, x_j)|}{\sum_{ij} |d(x_i, x_j)| \log\left(|d(x_i, x_j)| \Big/ \sum_{ij} |d(x_i, x_j)| \right)} \tag{6}$$

The value of information *J(ρ)* is a measure of equality of numbers among themselves. Applying the same formalism of MaxEnt Principle it is easy to define the MaxInf Principle on the basis of so called *Laplace's Principle* of insufficient knowledge[5][6]. *MaxInf Principle: <u>Out of all knowledge, choose the solution nearest to the uniform distribution of information.</u>* In the situations in which there are no reasons to prefer a solution, it is better choose the solution with uniform distribution, or one close to the uniform distribution of information.

3. AXIOMATIC DESIGN FOUNDAMENTALS

Nam P. Suh (1990) proposes an axiomatic method for highly complex designs. The design process optimizes elements using a set $\{FR_i\}$ of functional requirements and a set $\{DP_j\}$ of physical parameter [8, 9].

For comparing two design, on the basis of Suh axioms, one can compare the information content of the two design which can satisfy the functionally parameters. The information content can be described by means similar to the Wiener-Shannon's theory [4].
When the number of DP_s is less then the number of FR_s, then the coupled design cannot be satisfied. Suppose that there is a set of three $\{FR_1, FR_2, FR_3\}$ and a set of two $\{DP_1, DP_2\}$, then the equation in matrix notation is

$$\begin{Bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \tag{7}$$

4. DESIGN WITH NUMBER OF DP_S LOWER THAN NUMBER OF FR_S

From *MaxInf* it's possible to find the max value for *J* when

$$J_1 = J_2 = \ldots = J_n; \frac{1}{|FR_1 - f(DP_1)|} = \ldots\ldots = \frac{1}{|FR_i - f(DP_i)|} \quad \forall i \tag{8}$$

The max of information is obtained when the approximating function $FR_i = f\left(DP_j\right)$ <u>*has the same error*</u> .

$$\left\|FR_1 - f(DP_1)\right\| = \ldots. = \left|FR_n - f(DP_n)\right| = \Delta FR_T \tag{9}$$

The deviations of $FR_i = f\left(DP_j\right)$ evaluated at certain abscissa and the given ordinate corresponding to the same abscissa:

$$FR_i - \sum_{ij} \frac{\partial FR_{ij}}{\partial DP_{ij}} DP_{ij} = \left|\Delta FR_T\right| \tag{10}$$

From the solution of the linear system $A\bar{x} = \bar{b}$ we have

$$\begin{Bmatrix} FR_1 \\ FR_2 \\ \vdots \\ FR_n \end{Bmatrix} = [A] \begin{Bmatrix} DP_1 \\ DP_2 \\ \vdots \\ \Delta FR_T \end{Bmatrix}$$

it's possible to calculate the value of ΔFR_T from which can be valuated the approximation with the max information.

5. APPLICATION TO FAILURE MODE ANALYSIS

The example, shown in this paper, is a case study extracted from table of the load conditions which cause failure mechanisms. The "mechanism" is a hollow shafting quill of a gearbox that works in severe conditions of steady-state temperature, of thermal stresses due to temperature oscillation and of vibration. For that kind of mechanism the failure mode analysis brings to define the rectangular Action matrix **[A]**, because four kind of failure can occur in that kind of load conditions. Translating that problem in Axiomatic terms It's possible to say that there are:

- four FRs (effects) that are to be fulfilled
- three DPs (causes) to control the behaviour of failure mode

The axiomatic system can be easily solved for obtaining the vector of solution *under tolerance*

$$\{DP_1, DP_2, DP_3, \Delta FR_T\}^T$$

6. CONCLUSION

Using the idea of information, in a larger way than the idea of probability, it is possible the formulation of an Extend Theory of Information for probabilistic and non-probabilistic events. When number of $DP_s < FR_s$, then the DP_s are insufficient to achieve all the FR_s. If is imposed to the domain of FR_s a set of tolerance, it is possible to carry out a mathematical transformations from which it is possible to obtain all lacking values of $DP_{S.}$ The solution, consistent with the values of constraints, is obtained selecting the solution that maximize the Wiener-Shannon information. In conclusion it is possible to assert that:

When number of $DP_s < FR_s$, using MaxEnt Principle, it is possible to obtain an approximate solution compatible with boundary conditions.

That concept was applied to a particular case of Design in which the design matrix represents the Action Matrix between causes that determine a failure and effect that are caused from them.

ACKNOWLEDGEMENT

This paper was written after several years of studies on Information theory and its extension to metric space, made by Prof. Pappalardo at University of Salerno, and use the axiomatic approach in a new field: the Failure Mode Analysis; that approach is very powerful because the Suh Axioms about the “better design” often work with rectangular matrixes and with information definition on probabilistic and non-probabilistic events.

REFERENCES

1. Jaynes E. T. Probability Theory: The logic of Science. http://omega. albany. edu: 8008/ Jaynes Book. html
2. J. Kampe de Feriet, Note di Teoria dell'Informazione, Anno Accademico 1971-72-Annali dell'Istituto di Matematica Applicata dell'Università di Roma. 1972.
3. N. Kapur, K. Kesavan, Entropy Optimization Principles, Acad. Press Inc. Boston 1992.
4. Naddeo, Axiomatic design of a concept of car-platform for an electrical rear-wheel drive vehicle: a comparison with a fuzzy approach, proceedings of ICAD 2004 International Conference on Axiomatic Design, 21-24 of June 2004, Seoul – South Korea, also published on proceedings of Convegno Nazionale XIV ADM – XXXIII AIAS, Innovazione nella Progettazione Industriale, 31/08 – 02/09 2004, Bari - Italy
5. Pappalardo, M. et al. - Measure of independence in soft design- Journal of Material Processing Technology – 124 (2002)-ELSEVIER.
6. M. Pappalardo, Information in Design Process, 2nd International Conference on Planned Maintenance, Reliability and Quality - Oxford, England 2nd -3rd April 1998.

7. M. Pappalardo, Information in Metric Space, proceedings of AMME 2002, Poland and on the way to be published by Elsevier Science – Journal of Material Processing Technology
8. Nam P. Suh: The Principles of Design. Oxford University Press New York 1990.
9. Nam P. Suh Axiomatic Design-. MIT-Pappalardo Series in Mechanical Engineering Oxford University Press New York 2001.
10. N. Wiener, Cybernetics, Paris Herman (Act. Sc. 1053) 1948.