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Investigation of flexibly vibrating subsystem of mechatronic system

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Analysis and modelling

ABSTRACT

Purpose: of this paper is to investigate the transients of characteristics of vibrating beams obtained by the exact and approximate methods and to answer to the question – if the method can be used to nominate the characteristics of mechatronic systems.

Design/methodology/approach: was to nominate the relevance or irrelevance between the characteristics obtained by considered methods – especially concerning the relevance of the natural frequencies-poles of characteristics of mechanical part of mechatronic system. The main subject of the research is the continuous vibrating beam.

Findings: this approach is fact, that approximate solutions fulfill all conditions for vibrating beams and some conditions only, particularly for vibrating beams as the subsystems of mechatronic systems.

Research limitations/implications: is that linear continuous flexibly vibrating beam is considered.

Practical implications: of this study is the main point is the analysis and the examination of flexibly vibrating discrete-continuous mechatronic systems which characteristics can be nominated with approximate methods only.

Originality/value: of this approach relies on the comparison of the compatibility of the characteristics of the mechatronic and mechanical systems with demanded accuracy, nominated with approximate method.

Keywords: Analysis and modeling; Applied Mechanics; Exact and approximate methods; Continuous system, Vibrating beam

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1. Introduction

In the Gliwice research Centre the problems of analysis of vibrating beam systems, discrete and discrete-continuous mechanical systems by means the structural numbers methods modelled by the graphs, hypergraphs[1] has been made in (e.g.[1,2,7]). The continuous-discrete torsionally vibrating mechatronic systems[2] were considered in [3]. The approximate method of analysis called Galerkin's method has been used to obtain the frequency-modal characteristics. To comparison of obtaining dynamical characteristics – dynamical flexibilities only for mechanical subsystem torsionally vibrating bar, as a part of

[1] Other diverse problems have been modeled by different kind of graphs next they were examined and analyzed in (e.g.[15]). The problems of synthesis of selected class of continuous, discrete-continuous and discrete mechanical systems concerning the frequency spectrum has been made

[2] The problems concerned of piezoelectricity and electrostriction were presented for example in [6, 8-14].

complex mechatronic system, exact method and Galerkin's method were used [5]. In this paper frequency analysis and frequency – modal analysis have been presented for the mechanical part of mechatronic system, that means flexibly vibrating beam. In this aim three methods - the exact one and two approximate ones of analysis - have been used.

2. Vibration beam as the subsystem of mechatronic system

2.1. Natural frequency analysis

The beam - as the subsystem of mechatronic system[3] (Fig. 1) - with the constant cross section, clamped on left end and free on the right one with harmonic force excitation in form $P(t) = P_0 \sin \omega t$ is considered.

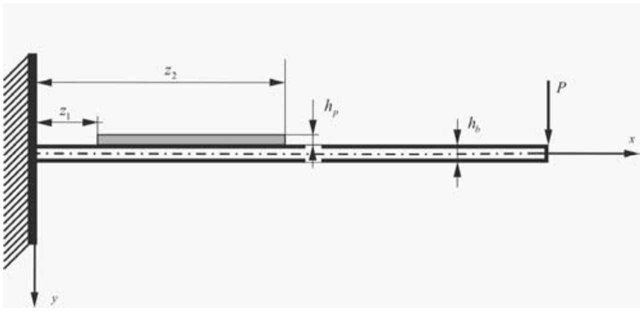


Fig. 1. Flexibly vibrating mechatronic system

The equation of motion of the beam only (Fig. 1) takes form

$$EIy(x,t)_{,xxxx} + \rho A y(x,t)_{,tt} = 0\,, \tag{1}$$

where: $y(x,t)$ - deflection at the time moment t of the lining beam section within the distance x from the beginning of the system, E - Young modulus, ρ - mass density of material of the beam, I - polar inertia moment of the beam cross section, A – area of the beam cross section.

The boundary conditions on the beam ends are following

$$y(0,t)=0,\ y_{,x}(0,t)=0,\ y_{,xx}(0,t)=0, EIy_{,xxx}(0,t)=-P(t), \tag{2}$$

where: l- length of the beam.

Own question (near homogeneous boundary conditions) for beam is however following

$$X^{(\mathrm{IV})}(x) - k^4 X(x) = 0, \tag{3}$$

$$X(0,t)=0, X^{'}(0,t)=0, X^{''}(l,t)=0, X^{''}(l,t)=0. \tag{4}$$

[3] The mechatronic system was considered in [4].

The general solution of own functions has the form

$$X(x) = A\sin kx + B\cos kx + C\sinh kx + D\cosh kx. \tag{5}$$

After substitution of following derivatives of (5) into boundary conditions (4) was received

$$X(0,t)=0, X^{'}(0,t)=0, X^{''}(l,t)=0, X^{''}(l,t)=0. \tag{6}$$

Out of set (6) results, that

$$\cos z = \frac{-1}{\cosh z},\quad z = kl. \tag{7}$$

The solution of equation (7) the own values are

$$z_n \approx \frac{2n-1}{2}\pi. \tag{8}$$

Relationships between constants A, B, C, D are following

$$B_n = A_n \frac{\cos z_n + \cosh z_n}{\sin z_n - \sinh z_n},\ C_n = -A_n,\ D_n = -A_n \frac{\cos z_n + \cosh z_n}{\sin z_n - \sinh z_n}, \tag{9}$$

and therefore own functions have form

$$X_n = A_n \begin{pmatrix} \sin \frac{z_n}{l} x + \frac{\cos z_n + \cosh z_n}{\sin z_n - \sinh z_n} \cos \frac{z_n}{l} x - \\ -\sinh \frac{z_n}{l} x - \frac{\cos z_n + \cosh z_n}{\sin z_n - \sinh z_n} \cosh \frac{z_n}{l} x \end{pmatrix},\ n = 1, 2, 3, \ldots . \tag{10}$$

2.2. The exact method of determining of dynamical flexibility

Deflection $y(x,t)$ is the harmonic function because the excitation is harmonic one, that means

$$y(x,t) = X(x)\sin \omega t\ . \tag{11}$$

Calculating suitable derivatives of (10) as well as substituting into (2) the set of equations, after transformations, was obtained

$$\begin{cases} B + D = 0 \\ A + C = 0 \\ -A\sin kl - B\cos kl + C\sinh kl + D\cosh kl = 0 \\ -A\cos kl + B\sin kl + C\cosh kl + D\sinh kl = \dfrac{-P_0}{EIk^3} \end{cases} \tag{12}$$

After transformations the set (10) is following

$$\begin{vmatrix} -(\cos kl + \cosh kl) & -(\sin kl + \sinh kl) \\ \sin kl - \sinh kl & -(\cos kl + \cosh kl) \end{vmatrix} \cdot \begin{vmatrix} A \\ B \end{vmatrix} = \begin{vmatrix} 0 \\ -\dfrac{P_0}{EIk^3} \end{vmatrix} =$$

$$= \mathbf{WA} = \mathbf{F}. \tag{13}$$

The main determinant of set of equations (13) equals

$$|\mathbf{W}| = \begin{vmatrix} -(\cos kl + \cosh kl) & -(\sin kl + \sinh kl) \\ \sin kl - \sinh kl & -(\cos kl + \cosh kl) \end{vmatrix} =$$

$$= 2(1 + \cos kl \cosh kl) \,. \tag{14}$$

To qualify constants *A, B, C, D* should count following determinants

$$\mathbf{W}_A = \begin{vmatrix} 0 & -(\sin kl + \sinh kl) \\ -\dfrac{P_0}{EIk^3} & -(\cos kl + \cosh kl) \end{vmatrix} = \frac{P_0}{EIk^3}(\sin kl + \sinh kl), \tag{15}$$

$$\mathbf{W}_B = \begin{vmatrix} -(\cos kl + \cosh kl) & 0 \\ -(\sin kl + \sinh kl) & -\dfrac{P_0}{EIk^3} \end{vmatrix} = \frac{P_0}{EIk^3}(\cos kl + \cosh kl) \,. \tag{16}$$

On the base (12-16) the constants *A*, … ,*D* are equal

$$A = -C = \frac{|\mathbf{W}_A|}{|\mathbf{W}|} = \frac{P_0(\sin kl + \sinh kl)}{2EIk^3(1 + \cos kl \cosh kl)} \,, \tag{17}$$

$$B = -D = \frac{|\mathbf{W}_B|}{|\mathbf{W}|} = \frac{P_0(\cos kl + \cosh kl)}{2EIk^3(1 + \cos kl \cosh kl)} \,. \tag{18}$$

Substituting expression (17) and (18) to (11) and taking into account (10) deflection beam is

$$y(x,t) = -\left[\frac{(\sin kl + \sinh kl)(\sin kx + \sinh kx)}{2EIk^3(1 + \cos kl \cosh kl)} + \right.$$

$$\left. = + \frac{(\cos kl + \cosh kl)(\sin kx + \sinh kx)}{2EIk^3(1 + \cos kl \cosh kl)}\right] P_0 \sin \omega t \,. \tag{19}$$

According to definition of dynamic flexibility, on the basis of (18), it takes form

$$Y = \frac{(\sin kl + \sinh kl)(\sin kx + \sinh kx) - (\cos kl + \cosh kl)(\sin kx + \sinh kx)}{2EIk^3(1 + \cos kl \cosh kl)} \,. \tag{20}$$

The transient of expression (20) is shown in Fig. 2a and the transient of absolute value of dynamic flexibility for *x=l*, that means $\alpha_Y = |Y|$ is drawn in Fig. 2b.

2.3. The orthogonalization method of determining of dynamical flexibility

The deflection *y(x,t)* is a harmonic function, that it is considered to be:

$$y(x,t) = \sum_{n=1}^{\infty} S_n(t) X_n(x), \tag{21}$$

where $S_n(t) = \dfrac{1}{\gamma_n^2}\displaystyle\int_0^l y(x,t) X_n(x)dx$, $\gamma_n^2 = \displaystyle\int_0^l X_n^2(x)dx =$

$$= \int_0^l \left(\sin\frac{z_n}{l}x + \frac{\cos z_n + \cosh z_n}{\sin z_n - \sinh z_n}\cos\frac{z_n}{l}x + \sinh\frac{z_n}{l}x + \right.$$

$$\left. + \frac{\cos z_n + \cosh z_n}{\sin z_n - \sinh z_n}\cosh\frac{z_n}{l}x \right)^2 dx \,,$$ z_n - are roots of equation (7) in form (8).

In result of ortogonalization of equation movement beam in form (1) it was received

$$EI\left[y_{,xxx} X_n - y_{,xx} X_n' + y_{,x} X_n'' - y X_n''' \right]_0^l + EI\int_0^l y(x,t) X_n^{(IV)} dx +$$

$$+ \rho A \int_0^l y_{,tt} X_n dx = 0. \tag{22}$$

Taking into consideration the boundary conditions (2) and the conditions of case of own function (10), it was received was

$$\ddot{S}_n + \omega_n^2 S_n = -\frac{P(t)}{\rho A \gamma_n^2} = -\frac{P_0}{\rho A \gamma_n^2}\sin \omega t. \tag{23}$$

The solution of equation (23) is following

$$S_n(t) = -\frac{P_0}{\rho A \gamma_n^2}\frac{1}{\omega_n^2 - \omega^2}\sin \omega t, \tag{24}$$

Deflection of beam is equal

$$y(x,t) = -\frac{P_0}{\rho A}\sum_{n=1}^{\infty}\frac{X_n(x)}{\gamma_n^2\left(\omega_n^2 - \omega^2\right)}\sin \omega t = \sum_{n=1}^{\infty} Y_n \sin \omega t. \tag{25}$$

On the basis (25) the dynamic flexibility is given as

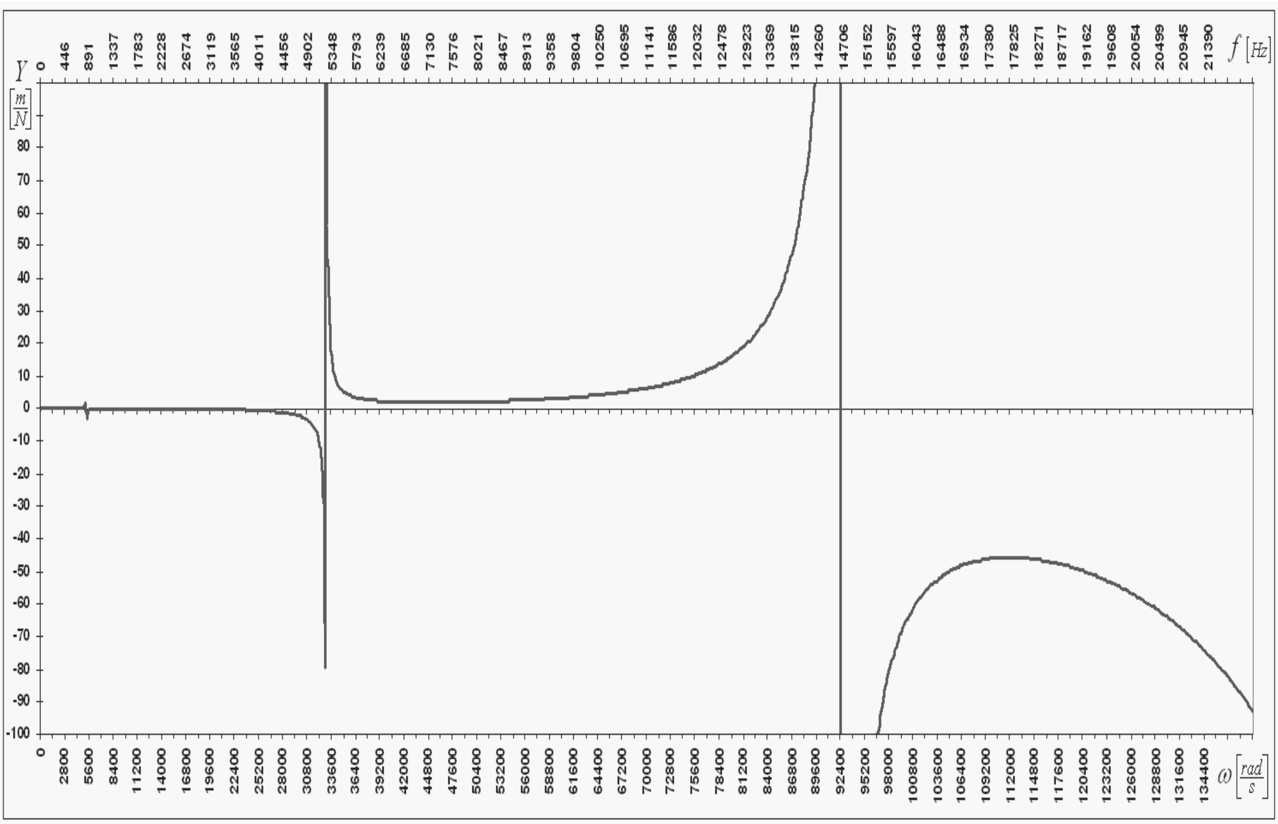
$$Y_n = \frac{X_n(x)}{\rho A \gamma_n^2 (\omega^2 - \omega_n^2)} \tag{26}$$

2.4. Galerkin's method of calculation of the dynamical flexibility of the beam

It has to be considered that if the shaft is under the action of moment with continuous factorization threw the beam length with the value $F(x)\sin\omega t$ on the length unit – then the equation of motion of the element with length dx lining in the point x is:

$$EIy_{,xxxx}dx + \rho A y_{,tt}dx = F(x)\sin \omega t dx \,. \tag{27}$$

a)



b)

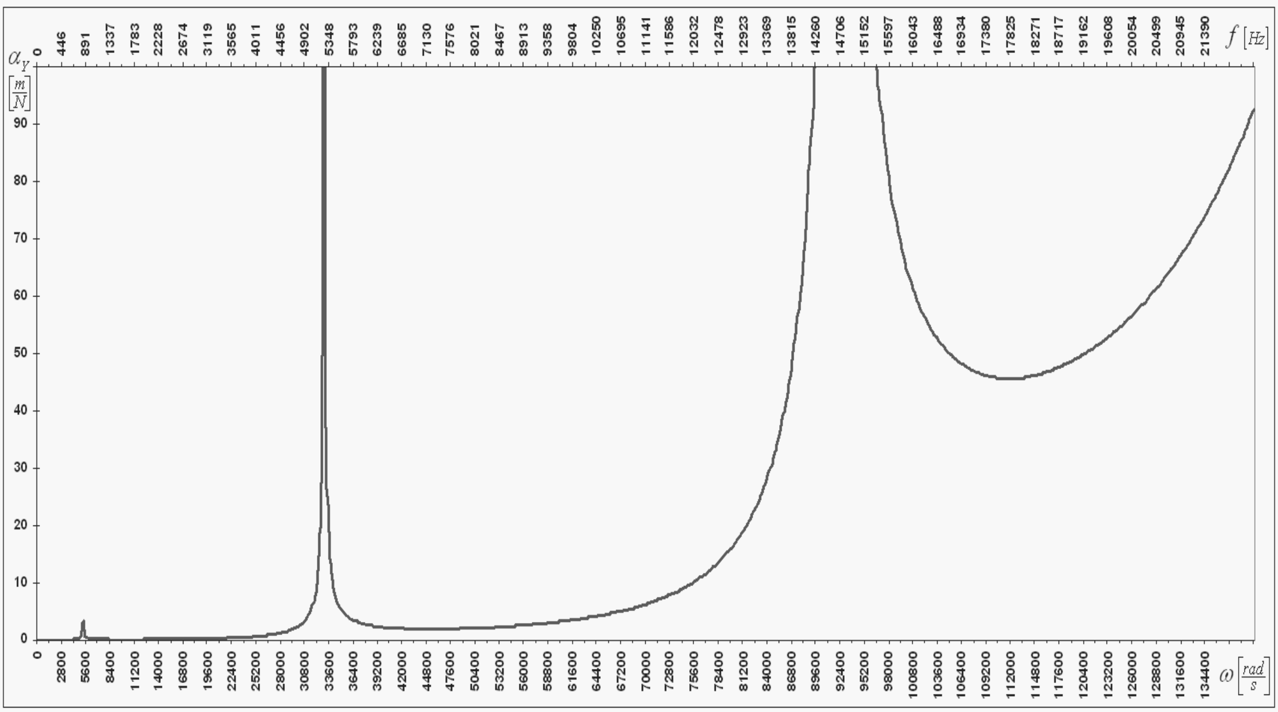


Fig. 2. The plot of dynamical flexibility of flexibly vibrating continuous system (a), transient of absolute value of dynamical flexibility (b)

To determine the dynamic flexibility the factors, which are compatible to concentrate loading $F\sin\omega t$, which works in point z have to be found. The loading can be considered as a limit of concentrate loading threw the length- as follows:

$$F(x)=\begin{cases}\frac{F}{h} & \text{when } z-h\le x\le z,\\ 0 & \text{in other section,}\end{cases} \tag{28}$$

and the equation of excitated vibrations of beam can described as

$$EIy'_{xxxx}+\rho Ay'_{tt}=P_0\sin\omega t, \tag{29}$$

where: $P_0=\frac{F}{h}$.

The defelection of beam - the solution of (29) by means Galerkin's method is given in shape of

$$y(x,t)=\sum_{n=1}^{\infty}y_n(x,t)=\sum_{n=1}^{\infty}A_n\sin\left[(2n-1)\frac{\pi}{2l}x\right]\sin\omega t. \tag{30}$$

Substituting the following derivative of expression (30) to (29) is obtained

$$EIA_n\left[(2k-1)\frac{\pi}{2l}\right]^4\sin\left[(2k-1)\frac{\pi}{2l}x\right]\sin\omega t+$$
$$+\rho AA_n\omega^2\sin\left[(2k-1)\frac{\pi}{2l}x\right]\sin\omega t=P_0\sin\omega t. \tag{31}$$

After transformations, the amplitude value A_n of the vibrations takes form of

$$A_n=\frac{P_0}{\rho A-EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{32}$$

Using the equation (32) and putting it to (30) the dynamical flexibility equals

$$Y_{xl}^{(n)}=\frac{\sin\left[(2n-1)\frac{\pi}{2l}x\right]}{\rho A\omega^2-EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{33}$$

It simply notices if in expression (26) to substitute own function, so as in Galerkin's method, it is received (33).

The absolute value of dynamical flexibility for the first vibration mode at the end of the beam, i.e. when $x=l$ takes the following form

$$\alpha_Y^{(1)}=\left|Y_{ll}^{(1)}\right|=\left|\frac{1}{\rho A\omega^2-EI\left(\frac{\pi}{2l}\right)^4}\right|. \tag{34}$$

The plot of expression (34) is shown in Fig. 3.

For the second vibration mode, i.e. when $n=2$, the dynamical flexibility (34) takes the form of

$$\alpha_Y^{(2)}=\left|Y_{ll}^{(2)}\right|=\left|\frac{1}{\rho A\omega^2-EI\left(\frac{3\pi}{2l}\right)^4}\right|. \tag{35}$$

The plot of expression (35) is shown in Fig. 4.

For the third vibration mode, i.e. when $n=3$, the characteristic (34) is given in shape

$$\alpha_Y^{(3)}=\left|Y_{ll}^{(3)}\right|=\left|\frac{1}{\rho A\omega^2-EI\left(\frac{5\pi}{2l}\right)^4}\right|. \tag{36}$$

The plot of equation (36) is shown in Fig. 5.

In global case the dynamical flexibility at the end of the beam gets shape of

$$Y_{xl}=\sum_{n=1}^{\infty}Y_{xl}^{(n)}=\sum_{n=1}^{\infty}\frac{\sin\left[(2n-1)\frac{\pi}{2l}x\right]}{\rho A\omega^2-EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{37}$$

For sum $k=1,2,3$ the plot of value of dynamical flexibility defined by expression (37) is shown in Fig. 6.

3. Last remark

On the base of the obtained formulas, which were determined by the exact method and approximate methods, it is possible to make the analysis of the considered class vibrating mechatronic systems. Moreover the analysis of mechatronic systems were the mechanical parts are vibrating beams it possible using only approximate methods.

In case of others of boundary conditions of mechanical parts of mechatronic systems that means the beam it is necessary to achieve offered researches in this paper. In future research works the problems shall be discussed.

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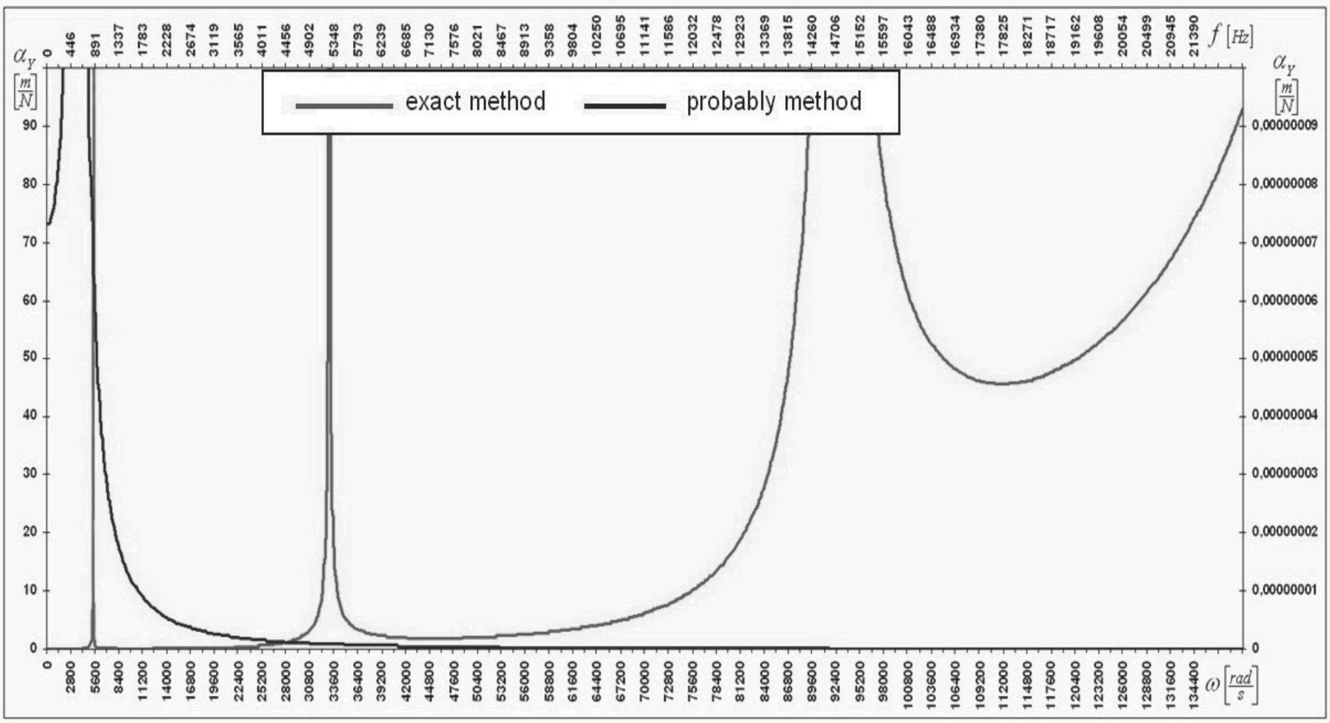

Fig. 3. The plot of absolute value of dynamical flexibility for the first mode vibration

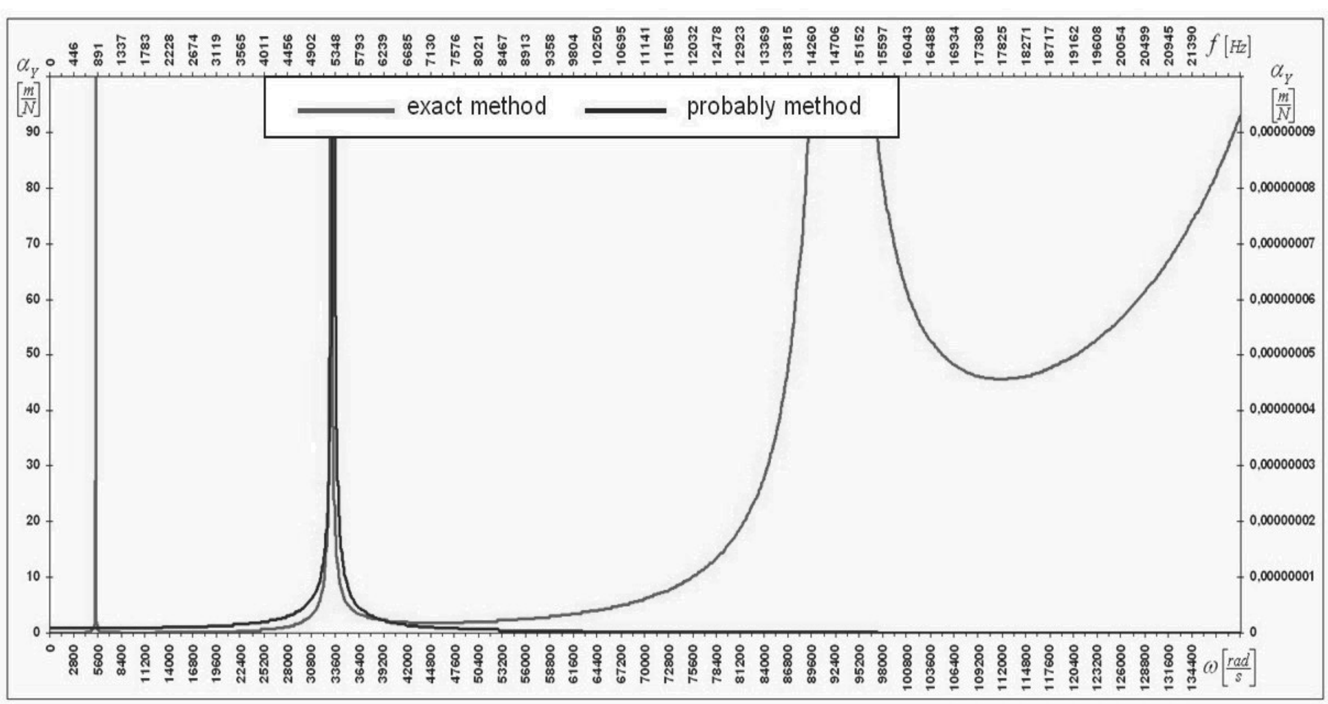

Fig. 4. The plot of absolute value of dynamical flexibility for the second mode vibration

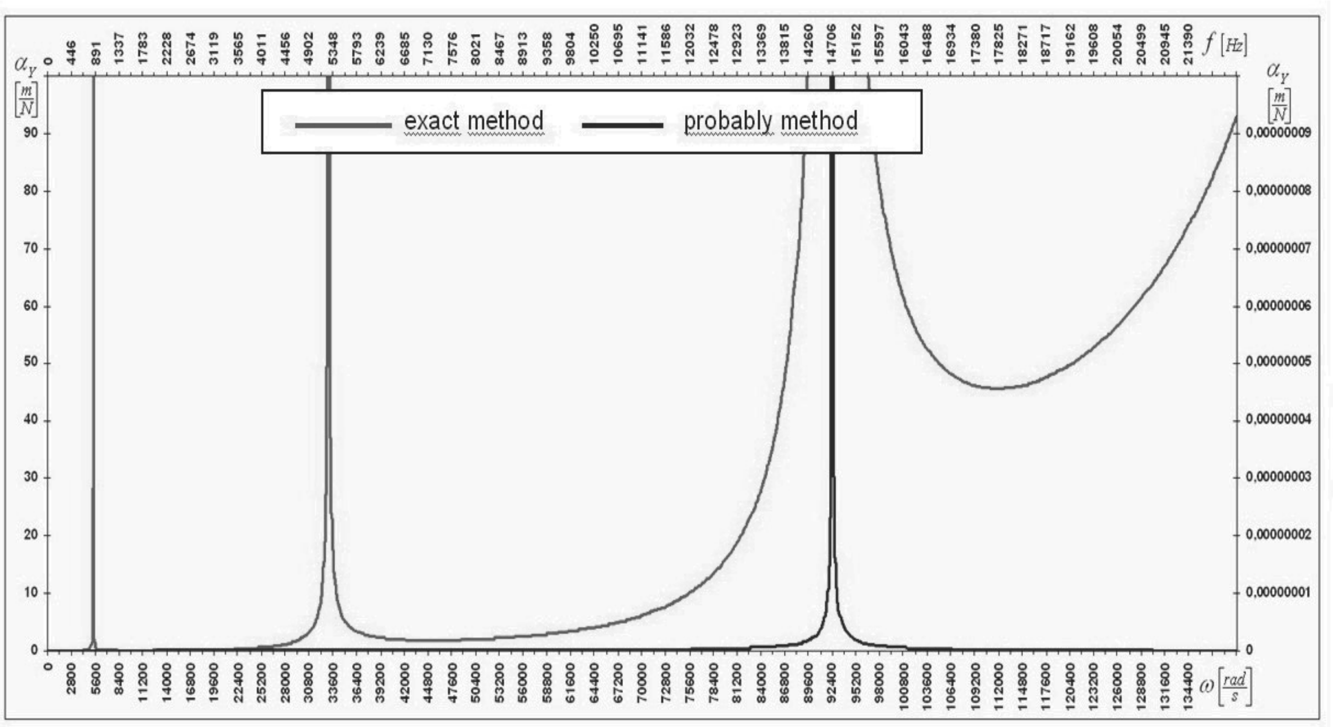

Fig. 5. The plot of absolute value of dynamical flexibility for the third mode vibration

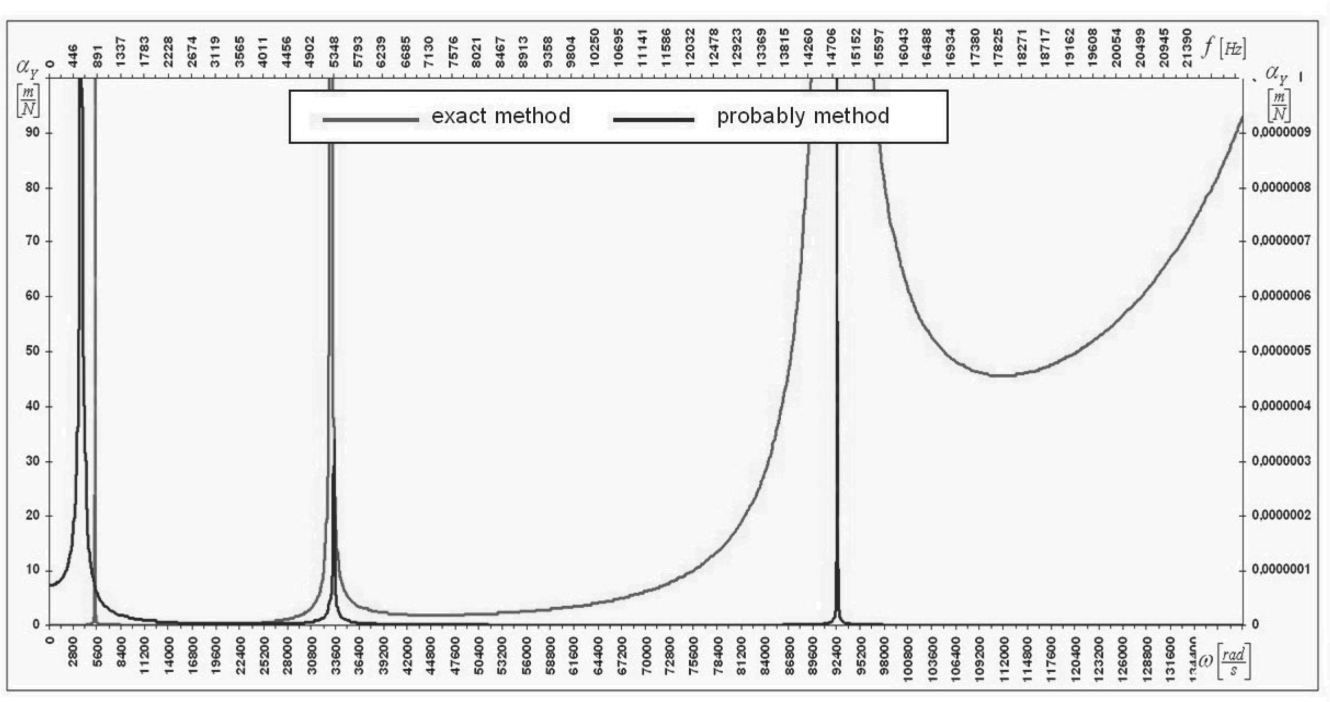

Fig. 6. The plot of absolute value of dynamical flexibility of the sum for n=1, 2, 3 mode vibration

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