

# Active synthesis of multiaxial drive systems using a comparative method

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## Analysis and modelling

### ABSTRACT

**Purpose:** of this paper is the problem of stability and vibration reduction in designed drive systems, in which the essential requirement is to meet the desired dynamic properties.

**Design/methodology/approach:** The method of stabilisation and reduction has been based on active synthesis, which makes it possible to obtain the desired mechanical effect through the proper selection of dynamic properties of the system, including the calculation of the active force as a function of the system force feedback.

**Findings:** Presented approach simplifies the process of selecting the dynamical parameters of machine drive systems in view of their dynamical characteristics.

**Research limitations/implications:** The scope of discussion is the synthesis of machine drive systems as discrete models of torsional vibrations. Such vibrations are more difficult to detect than flexural ones, which are accompanied by noise and vibrations of the adjacent elements (for example, shaft frames). Due to the absence of symptoms, torsional vibrations are particularly dangerous, as they may be unnoticeable until the destruction of subsystems occurs.

**Practical implications:** High durability and reliability of drive systems is associated with proper setting of system parameters - inertial, elastical and damping. Proper setting of these parameters is made possible by applying synthesis techniques.

**Originality/value:** We should emphasize that the considered problem varies from other issues met in classic mechanics or control theory. The research has been undertaken on the basis of topological methods, developed in scholar environment of Gliwice.

**Keywords:** Constructional design; Dynamic flexibility; Mechanical systems; Vibration reduction

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## 1. Introduction

Nowadays, machines are faced with increasing requirements as to their manufacturing properties, durability, energy-efficiency and safety, not to mention their hushed and steady operation mode. One of the most fundamental criteria in the design of modern mechanical structures are their dynamical properties, as they have a direct impact on the vibrations of the system, noise

emission, fatigue resistance, controllability and stability. Technological advancement forced machines designers to provide a high level of durability and reliability under operating conditions. Therefore, potential issues that may cause disturbances in the operation of machines, leading to drastic deterioration of work conditions, need to be addressed as early as at the design stage. The selection of the dynamical properties of machines is one of the methods enhancing their durability and

reliability. Such task may be accomplished with the use of the analysis [10-22] and synthesis algorithm [1-9,23,24].

Operating a machine near its resonant condition is possible only when the value of damping of the said system is large enough, due to the most significant tensions. Damping plays a decisive role in these cases, as it significantly reduces the amplitude of vibrations. The reduction of vibrations in the system can be achieved at both the design level and through adaptation of existing machinery with regard to the requirements of production processes. There are many methods and technologies allowing a reduction of unwanted vibrations of the machine [1,5,8,10,12-14, 21,22]. The methods can be divided into the following groups: passive vibration reduction, active vibration reduction and semi-active vibration reduction. The usual method of vibration reduction is through the introduction of dampers (passive vibration reduction) to the object, their task being to increase energy dispersion in the system. The characteristic feature of the resultant mechanical systems is that the set parameters of the system do not change over time, as well as the fact that achieving the set properties does not require an external energy source. Previous work by the authors focused on passive synthesis of systems understood in such a way, i.e. a calculation method, by which a mechanical system with parameters is designed to meet the desired characteristics in the form of chosen resonant and anti-resonant frequencies [1,5,8]. This resulted in the possibility of synthesis of discrete damped systems using models for viscous damping: proportional to inertial parameters (representing damping caused by the environment, such as air resistance and, in approximate terms, structural damping) and proportional to the stiffness (representing material damping), and combinations of the above (Rayleigh model).

Modern computer technology enables more effective methods of vibration reduction, which is active adjustment. This consists of determining the active force exerted on the system, which neutralises dynamic loads causing vibrations. This work will present the active synthesis of machine drive systems as models of torsional vibrations. Such vibrations are more difficult to detect than flexural ones, which are accompanied by noise and vibrations of the adjacent elements (for example, shaft frames). Due to the absence of symptoms, torsional vibrations are particularly dangerous, as they may be unnoticeable until the destruction of subsystems occurs. The objective of designing an active system is to ensure that the system meets the main conditions for correct operation. The primary criterion here is the stability of the system near the resonant condition of the system. The use of active synthesis, which makes it possible to obtain parameters and structures of systems in accordance with their dynamic characteristics, can be used as a tool for aiding design for all operating conditions. This can therefore be seen as a supporting step for the design of mechanical systems, in which the essential requirement is to meet the desired dynamic properties.

## 2. Synthesis of multi-axial drive systems

The first step in the synthesis of mechanical systems is the creation of mathematical functions, which on the one hand meet the conditions required for systems, and on the other can be

accurately performed in a real system. The method presented in this work for analytical determining of the dynamic characteristics is based on the adoption of the resonant and anti-resonant frequencies (poles and zeros of the desired dynamic characteristic).

The first step of active synthesis is setting the parameters and structure of the system for the characteristic shown in Fig. 1 - passive synthesis.

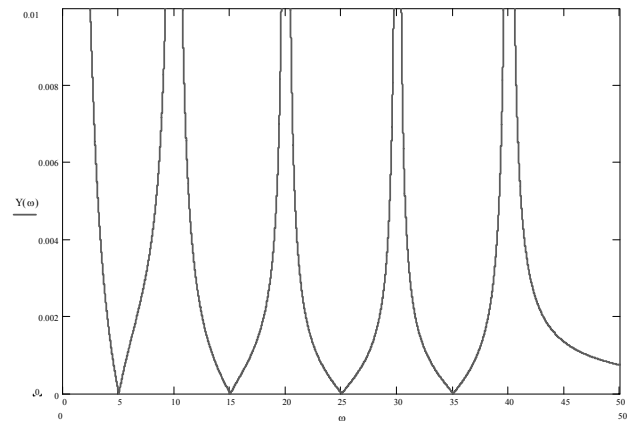


Fig. 1. Dynamic characteristics of a system subjected to passive synthesis

Therefore, if we take a frequency string in the form of:

- poles:

$$\omega_0 = 0 \text{ rad/s}, \quad \omega_2 = 10 \text{ rad/s}, \quad \omega_4 = 20 \text{ rad/s}, \quad \omega_6 = 30 \text{ rad/s}, \\ \omega_8 = 40 \text{ rad/s},$$

- zeros:

$$\omega_1 = 5 \text{ rad/s}, \quad \omega_3 = 15 \text{ rad/s}, \quad \omega_5 = 25 \text{ rad/s}, \quad \omega_7 = 35 \text{ rad/s},$$

then the function describing dynamic properties of the torsionally vibrating discrete system can be represented in the form of dynamic flexibility:

$$Y(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{s^2(s^2 + 10^2)(s^2 + 20^2)(s^2 + 30^2)(s^2 + 40^2)}, \quad (1)$$

or immobility (mechanical impedance):

$$U(s) = H \frac{s(s^2 + 10^2)(s^2 + 20^2)(s^2 + 30^2)(s^2 + 40^2)}{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}, \quad (2)$$

where:  $s = j\omega$ ,  $H$  - any positive real number. The characteristic function, in the form of impedance (2) was used to determine the structure and parameters of the model of the machine drive system. The branched structure of the proposed system, together with the values of the inertial and elastic elements, was obtained using the mixed method of synthesising of the dynamic characteristics (combining the continued fraction distribution method with the partial fraction distribution method - theoretical details and numerical examples of this method are presented in [1,4]) in the following form:

$$U(s) = J_1 s + \frac{1}{\frac{s}{c_2} + \frac{1}{J_2 s + \frac{1}{\frac{s}{c_3} + \frac{1}{J_3 s}}}} + \frac{1}{\frac{s}{c_4} + \frac{1}{J_4 s + \frac{1}{\frac{s}{c_5} + \frac{1}{J_5 s}}}}, \quad (3)$$

where:  $J_1 = 1$  [kgm<sup>2</sup>],  $J_2 = 4.28$  [kgm<sup>2</sup>],  $J_3 = 7.61$  [kgm<sup>2</sup>],  $J_4 = 0.43$  [kgm<sup>2</sup>],  $J_5 = 0.05$  [kgm<sup>2</sup>] - value of the inertial elements of the sought system,  $c_2 = 522.95$  [Nm/rad],  $c_3 = 350.71$  [Nm/rad],  $c_4 = 377.04$  [Nm/rad],  $c_5 = 43.1$  [Nm/rad] - values of elastic elements of the sought system. Figure 2 shows a discrete mechanical system as the physical realisation of the synthesised characteristic 2.

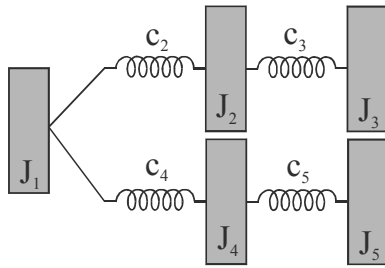


Fig. 2. A model of the drive system corresponding to the characteristic distribution (2)

After determining the parameters and structure of the passive system, it is possible to proceed to determine the force that will allow stabilising and reduction in vibrations of the system in the vicinity of the resonant state of the system. The implementation of the excitation force, as the setting value, can be made in the system, both for the first and other inertial elements. In considering the desired characteristics (1), the sought force is determined for the first inertial element - Fig.3.

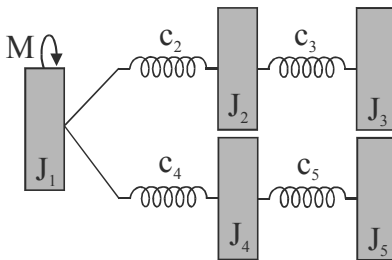


Fig. 3. A model of a drive system with active force

For this purpose, the dynamic characteristics (1) are modified, introducing the parameter  $h$  for the decrease in frequency of the chosen resonant frequency, in the form:

- for the one resonant frequency:

$$Y1(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 20^2)(s^2 + 30^2)(s^2 + 40^2)}, \quad (4)$$

and

$$Y2(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{s^2(s^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)(s^2 + 30^2)(s^2 + 40^2)}, \quad (5)$$

or

$$Y3(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{s^2(s^2 + 10^2)(s^2 + 20^2)(s^2 + 2h_3s + h_3^2 + 30^2)(s^2 + 40^2)}, \quad (6)$$

and

$$Y4(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{s^2(s^2 + 10^2)(s^2 + 20^2)(s^2 + 30^2)(s^2 + 2h_4s + h_4^2 + 40^2)}; \quad (7)$$

- for the two resonant frequencies:

$$Y5(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)}{(s^2 + 30^2)(s^2 + 40^2)} \right]}, \quad (8)$$

and

$$Y6(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 20^2)}{(s^2 + 2h_3s + h_3^2 + 30^2)(s^2 + 40^2)} \right]}, \quad (9)$$

or

$$Y7(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 20^2)}{(s^2 + 30^2)(s^2 + 2h_4s + h_4^2 + 40^2)} \right]}, \quad (10)$$

and

$$Y8(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)}{(s^2 + 2h_3s + h_3^2 + 30^2)(s^2 + 40^2)} \right]}, \quad (11)$$

or

$$Y9(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)}{(s^2 + 30^2)(s^2 + 2h_4s + h_4^2 + 40^2)} \right]}, \quad (12)$$

and

$$Y10(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 10^2)(s^2 + 20^2)(s^2 + 2h_3s + h_3^2 + 30^2)}{(s^2 + 2h_4s + h_4^2 + 40^2)} \right]}; \quad (13)$$

- for the three resonant frequencies:

$$Y11(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)}{(s^2 + 2h_3s + h_3^2 + 30^2)(s^2 + 40^2)} \right]}, \quad (14)$$

and

$$Y12(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)}{(s^2 + 30^2)(s^2 + 2h_4s + h_4^2 + 40^2)} \right]}, \quad (15)$$

Or

$$Y_{13}(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 20^2)}{(s^2 + 2h_3s + h_3^2 + 30^2)(s^2 + 2h_4s + h_4^2 + 40^2)} \right]}, \quad (16)$$

and

$$Y_{14}(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)}{(s^2 + 2h_3s + h_3^2 + 30^2)(s^2 + 2h_4s + h_4^2 + 40^2)} \right]}; \quad (17)$$

for the four resonant frequencies:

$$Y_{15}(s) = \frac{(s^2 + 5^2)(s^2 + 15^2)(s^2 + 25^2)(s^2 + 35^2)}{\left[ \frac{s^2(s^2 + 2h_1s + h_1^2 + 10^2)(s^2 + 2h_2s + h_2^2 + 20^2)}{(s^2 + 2h_3s + h_3^2 + 30^2)(s^2 + 2h_4s + h_4^2 + 40^2)} \right]}. \quad (18)$$

As a result of such modifications to the characteristics (2), a reduction in vibrations of the system in the area of the one resonant frequency is achieved - characteristics (4-7) or the two resonant frequencies - characteristics (8 - 13), there exists also a possibility of reducing vibrations in the area of three resonant frequencies - characteristics (14-17) and the four resonant frequencies - characteristic (18). An example of vibration reduction defined in such a way are shown in Figs. 4-7.

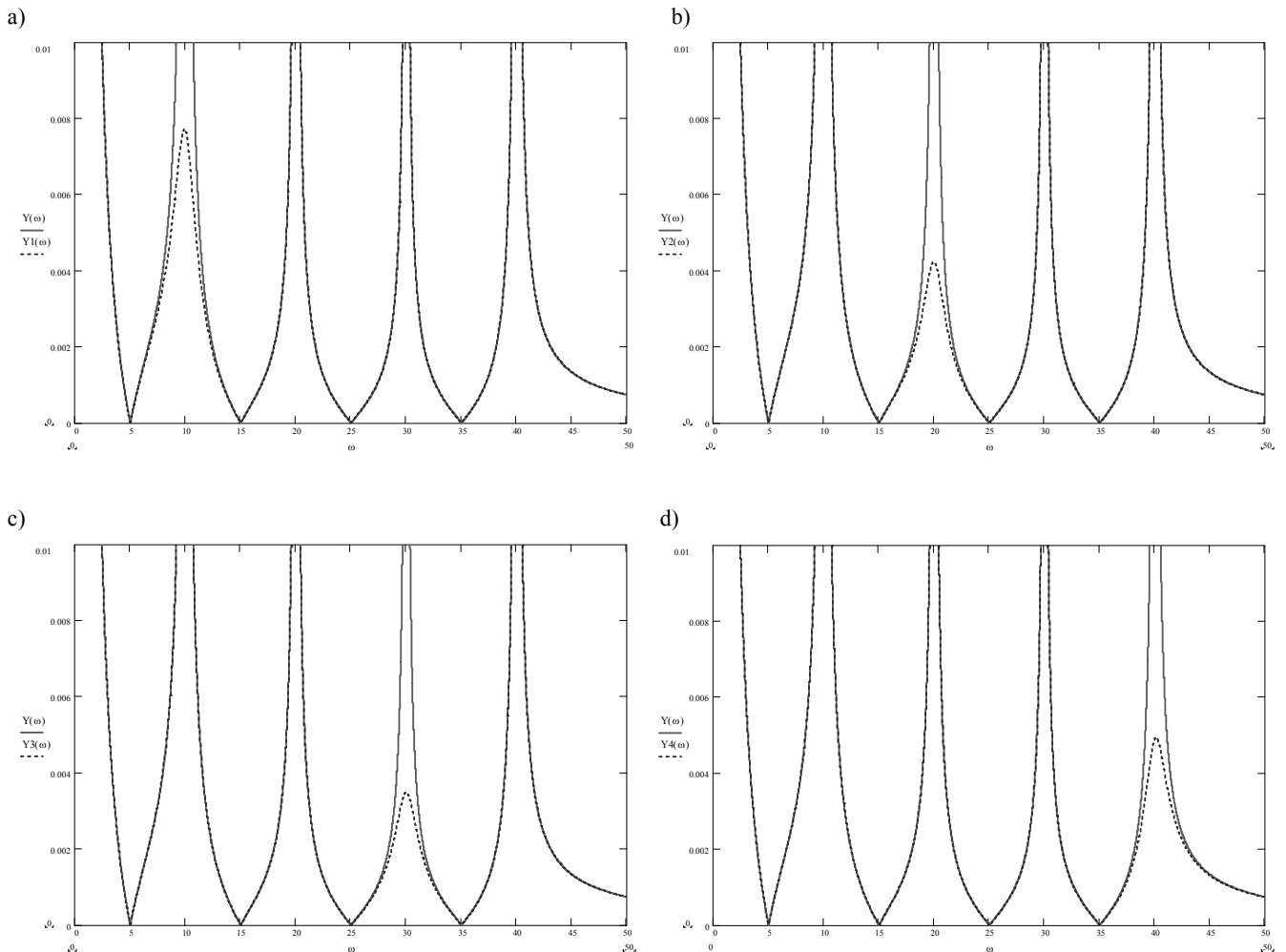


Fig. 4. Dynamic characteristics of a system subjected to active synthesis: a) reduction of the first resonant frequency of the system b) reduction of the second resonant frequency of the system c) reduction of the third resonant frequency of the system d) reduction of the fourth resonant frequency of the system

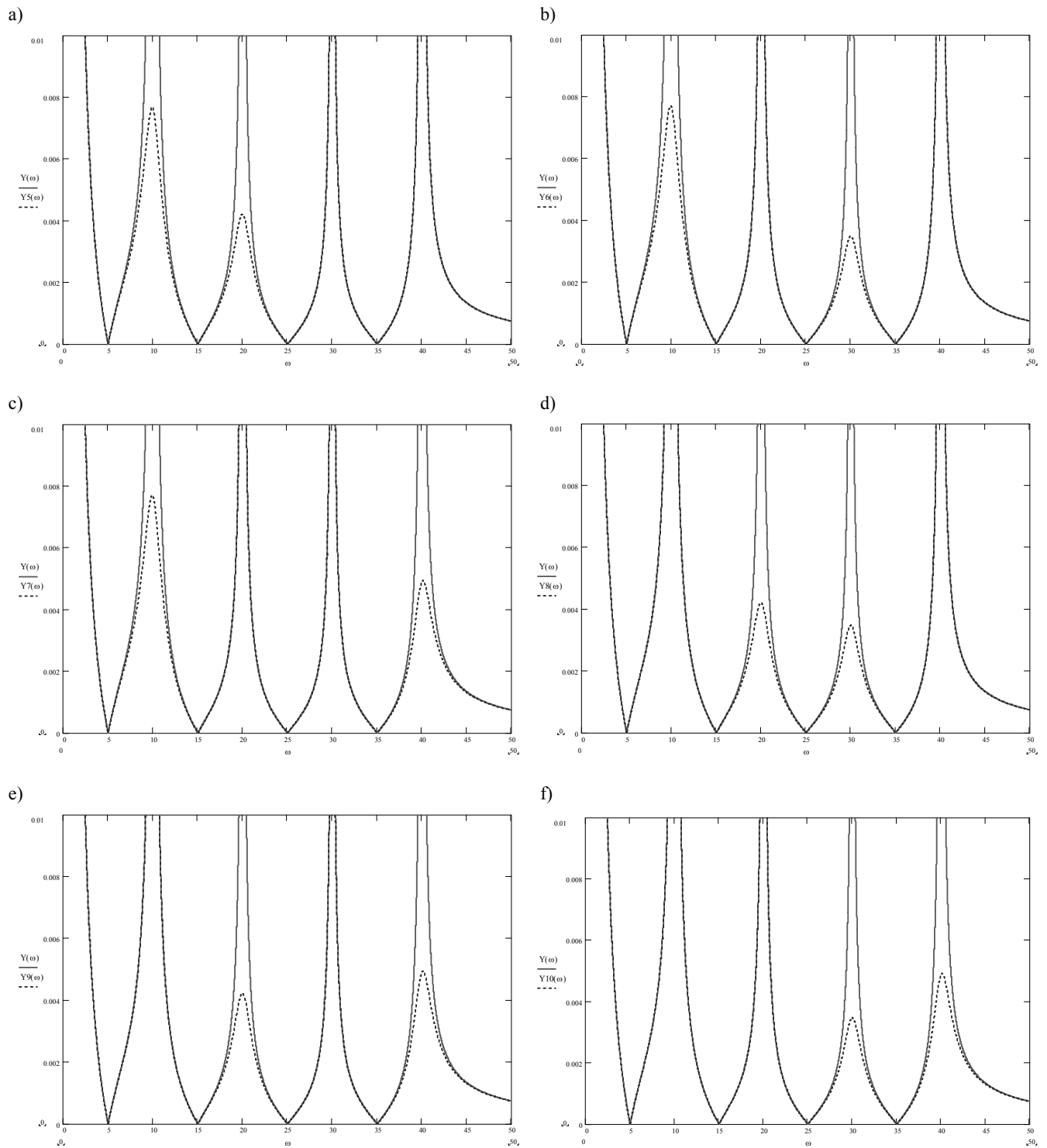


Fig. 5. Dynamic characteristics of a system subjected to active synthesis: a) reduction of the first and second resonant frequency of the system b) reduction of the first and third resonant frequency of the system c) reduction of the first and fourth resonant frequency of the system d) reduction of the second and third resonant frequency of the system e) reduction of the second and fourth resonant frequency of the system f) reduction of the third and fourth resonant frequency of the system

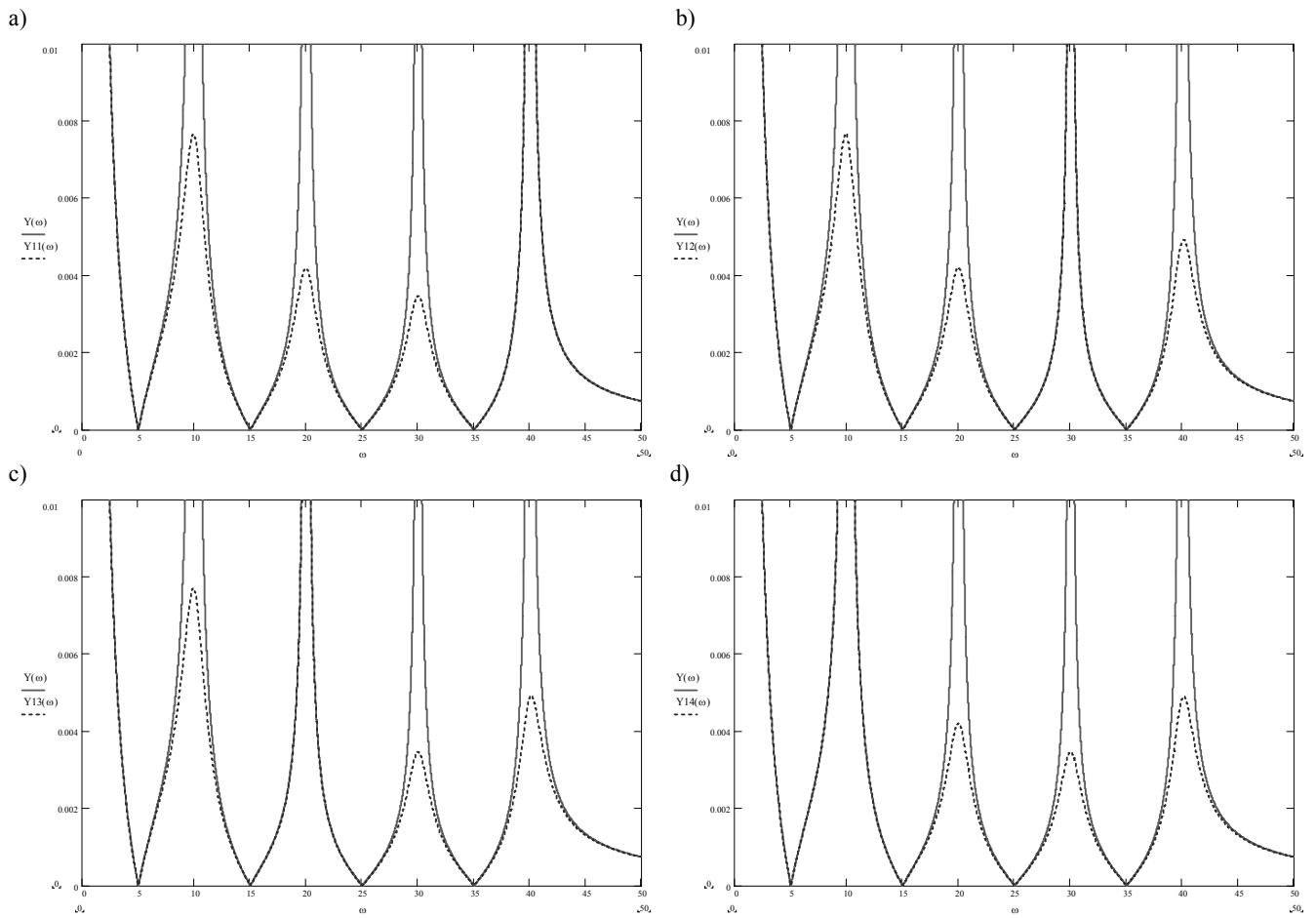


Fig. 6. Dynamic characteristics of a system subjected to active synthesis: a) reduction of the first, second and third resonant frequency of the system b) reduction of the first, second and fourth resonant frequency of the system c) reduction of the first, third and fourth resonant frequency of the system d) reduction of the second, third and fourth resonant frequency of the system

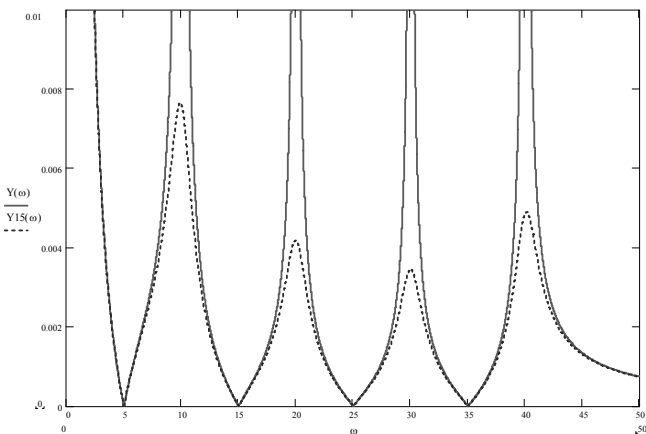


Fig. 7. Dynamic characteristics of a system subjected to active synthesis - reduction of the first, second, third and fourth resonant frequency of the system

A decline in the value of vibration amplitude depends on the introduced parameter of the decline in frequency of vibration - impact of changes in the  $h_1$  parameter value on the amplitude of the first resonant frequency is shown in Fig. 8.

In order to decrease the vibrations near the resonant frequency of the analysed system (Fig. 2), a control law is adopted, allowing the calculation of the excitation force as a function of the force feedback in the following form:

$$M = -(k_{p1}\varphi_1 + k_{v1}\dot{\varphi}_1 + k_{p2}\varphi_2 + k_{v2}\dot{\varphi}_2 + k_{p3}\varphi_3 + k_{v3}\dot{\varphi}_3 + k_{p4}\varphi_4 + k_{v4}\dot{\varphi}_4 + k_{p5}\varphi_5 + k_{v5}\dot{\varphi}_5), \quad (19)$$

where:  $k_{p1}, k_{p2}, k_{p3}, k_{p4}, k_{p5}, k_{v1}, k_{v2}, k_{v3}, k_{v4}, k_{v5}$  - coefficients of the gain of the control system dependant of the position and velocity of inertial elements of the analysed system.

In the following part, the method for calculating these coefficients has been shown, which in turn will permit the

determination of the control force. For this purpose a block diagram is built of a closed system including the controllers for the force inductors in the system, as shown in Fig. 9.

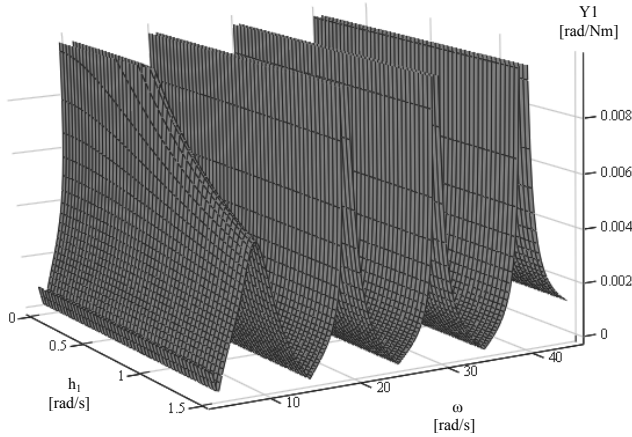


Fig. 8. Dynamic flexibility in the function of changes to parameter for frequency decline  $h_1$

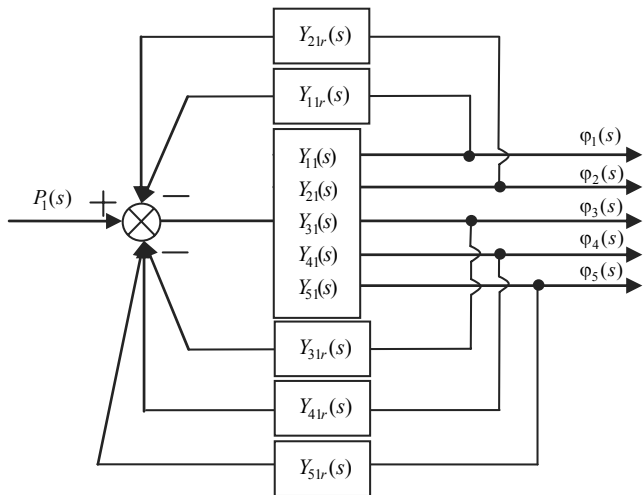


Fig. 9. Block diagram of the analysed control system

Dynamic flexibility between the first input and output of the system with the vibration eliminator in the form of a dynamic force is as follows:

$$Y1r(s) = \frac{Y_{11}(s)}{1 + Y_{11}(s) \cdot Y_{11r}(s) + Y_{21}(s) \cdot Y_{21r}(s) + Y_{31}(s) \cdot Y_{31r}(s) + Y_{41}(s) \cdot Y_{41r}(s) + Y_{51}(s) \cdot Y_{51r}(s)}, \quad (20)$$

where:

$$Y_{11}(s) = \frac{0.69s^8 + 1453.06s^6 + 853641.99s^4 + 139633226.48s^2 + 2979697918.17}{0.69s^{10} + 2075.83s^8 + 1888938.81s^6 + 567359495.87s^4 + 39853069315.06s^2}$$
 - the transfer function between the first input and output of the analysed system

in Fig. 2,  $Y_{21}(s) = \frac{84.46s^6 + 160136.78s^4 + 71858844.94s^2 + 2979697918.17}{0.69s^{10} + 2075.83s^8 + 1888938.81s^6 + 567359495.87s^4 + 39853069315.06s^2}$  -

transfer function between the second output and the first input of the analysed system;

$$Y_{31}(s) = \frac{3892.26s^4 + 7200309.21s^2 + 2979697918.17}{0.69s^{10} + 2075.83s^8 + 1888938.81s^6 + 567359495.87s^4 + 39853069315.06s^2}$$
 - transfer function between the third output and the first input of the analysed system;

function between the third output and the first input of the analysed system;

$$Y_{41}(s) = \frac{605.45s^6 + 681093.39s^4 + 135838513.53s^2 + 2979697918.17}{0.69s^{10} + 2075.83s^8 + 1888938.81s^6 + 567359495.87s^4 + 39853069315.06s^2}$$
 - transfer function between the fourth output and the first input of the analysed system;

function between the fourth output and the first input of the analysed system;

$$Y_{51}(s) = \frac{529731.39s^4 + 132432917.27s^2 + 2979697918.17}{0.69s^{10} + 2075.83s^8 + 1888938.81s^6 + 567359495.87s^4 + 39853069315.06s^2}$$
 -

transfer function between the fifth output and the first input of the analysed system;  $Y_{11r}(s) = k_{p1} + k_{v1}s$  - transfer function of the controller in the force feedback loop from the first movement;  $Y_{21r}(s) = k_{p2} + k_{v2}s$  - transfer function of the controller in the force feedback loop from the second movement;  $Y_{31r}(s) = k_{p3} + k_{v3}s$  - transfer function of the controller in the force feedback loop from the third movement;  $Y_{41r}(s) = k_{p4} + k_{v4}s$  - transfer function of the controller in the force feedback loop from the fourth movement;  $Y_{51r}(s) = k_{p5} + k_{v5}s$  - transfer function of the controller in the force feedback loop from the fifth movement.

The value of amplification factors of the control system is determined by comparing the characteristics (4-18) with the characteristic of dynamic flexibility of the control system in the force feedback loop (20). As a result of the comparison of characteristics, including expressions with equal powers and taking into account the gain in the characteristic of the analysed system (20), fifteen sets of equations have been determined, which will be used to determine the gain coefficients of the

control force. In the case of reduction of the first resonant frequency, it has the form:

$$\begin{aligned}
 & k_{v1} = 2h_1, \\
 & k_{p1} = h_1^2, \\
 & \frac{605.45k_{v4} + 84.46k_{v2} + 1453.06k_{v1}}{0.69} = 5800h_1, \\
 & \frac{605.45k_{p4} + 84.46k_{p2} + 1453.06k_{p1}}{0.69} = 2900h_1^2, \\
 & \left[ \frac{853641.99k_{v1} + 160136.78k_{v2} + 3892.26k_{v3} +}{681093.39k_{v4} + 529731.39k_{v5}} \right] = 4880000h_1, \\
 & \left[ \frac{853641.99k_{p1} + 160136.78k_{p2} + 3892.26k_{p3} +}{681093.39k_{p4} + 529731.39k_{p5}} \right] = 2440000h_1^2, \\
 & \left[ \frac{139633226.48k_{v1} + 71858844.94k_{v2} +}{+ 7200309.21k_{v3} + 135838513.53k_{v4} +}{+ 132432917.27k_{v5}} \right] = 1152000000h_1, \\
 & \left[ \frac{139633226.48k_{p1} + 71858844.94k_{p2} +}{+ 7200309.21k_{p3} + 135838513.53k_{p4} +}{+ 132432917.27k_{p5}} \right] = 576000000h_1^2, \\
 & \frac{2979697918.17(k_{v1} + k_{v2} + k_{v3} + k_{v4} + k_{v5})}{0.69} = 0, \\
 & \frac{2979697918.17(k_{p1} + k_{p2} + k_{p3} + k_{p4} + k_{p5})}{0.69} = 0,
 \end{aligned} \tag{21}$$

Assuming the decline in value of natural vibration frequency  $h_1 = 1$  [rad/s], the values of the control force have been set. The same way as in the case of reduction of the first resonant frequency, the values of the control force have been set for other cases of reduce vibrations (assuming  $h_i = 1$ ,  $i = 1,2,3,4$ ).

Table 1.  
Values of gain of the control system - reduction of the one resonant frequency

	Y1(s)	Y2(s)	Y3(s)	Y4(s)
$k_{v1}$	2	2	2	2
$k_{p1}$	1	1	1	1
$k_{v2}$	6.016	-5.641	-1.512	-0.751
$k_{p2}$	3.008	-2.821	-0.756	-0.375
$k_{v3}$	-9.133	1.305	0.145	3.955E-2
$k_{p3}$	-4.567	0.652	7.250E-2	1.977E-2
$k_{v4}$	0.990	1.930	0.211	-1.495
$k_{p4}$	0.495	0.965	0.106	-0.748
$k_{v5}$	0.128	0.406	-0.844	0.206
$k_{p5}$	6.387E-2	0.203	-0.422	0.103

Tables 1-3 summarises the gain values of the control force acting on the first inertial element.

After determining all cases of application of the control force on the first inertial element, one can proceed to determine the force acting on other inertial elements.

### 3. Analysis of results

Analysis of the results was carried out in the Matlab/Simulink package. Numerical verifications were carried out for the case when the system will be operating with the control force exerted determined for the characteristic (4) - Y1(s). In the simulation, the excitation signal was determined as a force with a unit amplitude and circular frequency corresponding to the resonant frequency of natural vibrations of the system. In addition, it was assumed that the control force is activated after 19 seconds. Angular displacements from the dynamic model are shown in Fig. 10.

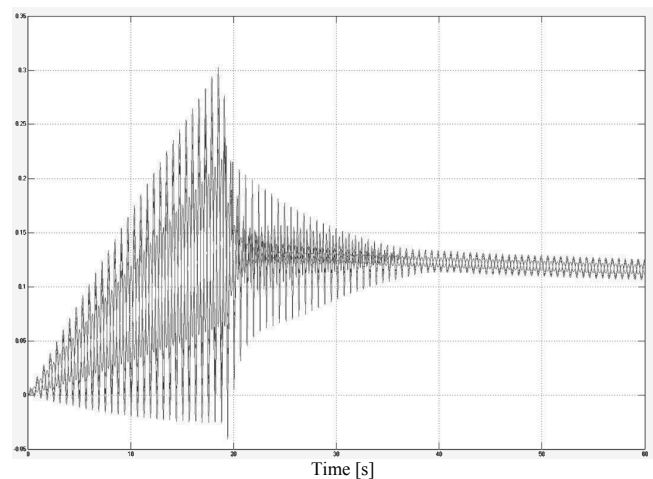


Fig. 10. Angular displacements of the system with excitation linked to the first resonant frequency of the system



Table 2.  
Values of gain of the control system - reduction of the two resonant frequencies

	Y5(s)	Y6(s)	Y7(s)	Y8(s)	Y9(s)	Y10(s)
$k_{v1}$	4	4	4	4	4	4
$k_{p1}$	6	6	6	6	6	6
$k_{v2}$	0.452	44.522	5.274	-7.170	-6.400	-2.265
$k_{p2}$	-18.848	-2.650	0.232	6.959E-4	-1.439	-0.674
$k_{v3}$	-7.898	-9.011	-9.106	1.454	1.346	0.185
$k_{p3}$	5.638	-1.890	-3.248	-0.840	-9.171E-2	-9.970E-2
$k_{v4}$	2.913	1.203	-0.502	2.148	0.441	-1.280
$k_{p4}$	5.945	0.829	-3.573	-1.255	-5.055	-8.019
$k_{v5}$	0.532	-0.714	0.334	-0.432	0.613	-0.641
$k_{p5}$	1.265	-2.288	0.590	-3.906	0.585	2.793

Table 3.  
Values of gain of the control system - reduction of the three and four resonant frequencies

	Y11(s)	Y12(s)	Y13(s)	Y14(s)	Y15(s)
$k_{v1}$	6	6	6	6	8
$k_{p1}$	15	15	15	15	28
$k_{v2}$	-1.114	-0.325	3.771	-7.926	-1.896
$k_{p2}$	-21.013	-19.908	-4.980	1.848	-21.601
$k_{v3}$	-7.744	-7.855	-8.980	1.494	-7.699
$k_{p3}$	6.790	6.212	-0.758	-1.779	7.167
$k_{v4}$	3.151	1.440	-0.274	0.672	1.701
$k_{p4}$	3.979	-3.376	-10.601	-14.640	-12.676
$k_{v5}$	-0.292	0.740	-0.517	-0.240	-0.106
$k_{p5}$	-4.756	2.071	1.339	-0.429	-0.891

## 4. Conclusions

The exertion of an active control force is tantamount to the provision of additional energy from the outside. When designing an active vibration reduction system, the values of gain of the control force should be determined so as to achieve the desired effect of reducing vibration at the smallest cost. The presented active synthesis makes it possible to meet criteria determined in such a way. This is due to the large number of gain parameters of the control force obtained through synthesis, which may substantially affect the optimal choice of control parameters.

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